Welcome to MAT137!

- Class will always begin at 9:10am.
- Hi! My name is Qin.
- Email: qin.deng@mail.utoronto.ca
- Office hours: WF12 - 1 on the lecture Zoom link
- Website for lecture slides: http://www.math.toronto.edu/dengqin/MAT137_S21.html
- Everything else will be posted on Quercus.
- **Homework:** Watch videos 1.7 - 1.15 before next lecture.
- **Problem set 1** will be posted soon. It is due on Friday May 14th at midnight.

What are the following sets?

1. $(2, 4] \cup (3, 5]$
2. $(-\infty, 4] \cap [3, \infty)$
3. $[4, 2)$
4. $(0, 0)$
5. $[0, 0]$

Set description

What are the following sets?

1. $\{x \in \mathbb{N} : x^2 < 6\}$
2. $\{x \in \mathbb{Z} : x^2 < 6\}$
3. $\{x \in \mathbb{R} : x^2 < 6\}$

Set description

What are the following sets?

1. $\{x \in \mathbb{R} : \forall y \in [0,1], x < y\}$
2. $\{x \in \mathbb{R} : \exists y \in [0,1] \text{ s.t. } x < y\}$
3. $\{x \in [0,1] : \forall y \in [0,1], x < y\}$
4. $\{x \in [0,1] : \exists y \in [0,1], x < y\}$
5. $\{x \in [0,1] : y \in [0,1], x < y\}$
6. $\{x \in [0,1] : \exists y \in \mathbb{R} \text{ s.t. } x < y\}$

New set operations: Set difference

Given two sets A and B. We define

$A \backslash B := \{x \in A : x \notin B\}$. This set is called “A minus B”.

What are the following sets?

1. $[0, 1] \backslash (-0.5, 1)$
2. $[0, 1] \backslash (1, \infty)$
3. $\mathbb{R} \backslash [0, 1]$
4. $[0, 1] \backslash \mathbb{R}$

Polling

1. $A := \{\text{Students currently in Ontario}\}$
2. $B := \{\text{Students who like cats more than dogs}\}$
3. $C := \{\text{Students who like math}\}$

Are you in $(A \backslash B) \cup (B \backslash A)$?

Polling

1. $A := \{\text{Students currently in Ontario}\}$
2. $B := \{\text{Students who like cats more than dogs}\}$
3. $C := \{\text{Students who like math}\}$

Are you in $C \backslash (B \backslash C)$?

Set description: even integers

Write a description of the set E of even integers using set-building notation.

Let S be the set of even integers. Which of the following is the correct set-building notation for S?

1. $\{x \in \mathbb{Z} : \forall n \in \mathbb{Z}, x = 2n\}$
2. $\{x \in \mathbb{Z} : \exists n \in \mathbb{Z} \text{ s.t. } x = 2n\}$

Which of these statements is true?

3. $\forall a \in \mathbb{Z}$, the number $n = 2a$ is even.
4. $\exists a \in \mathbb{Z}$ s.t. the number $n = 2a$ is even.

Let f be a function with domain $\mathbb{R}$. Rewrite the following statements using $\forall$ or $\exists$:

1. The graph of f intercepts the x-axis.
2. f is the zero function.
3. f is not the zero function.
4. f never vanishes.
5. The equation $f(x) = 0$ has a solution.
6. The equation $f(x) = 0$ has no solutions.
7. f takes both positive and negative values.
8. f is never negative.

Negation intro

The negation of a logic statement is a statement which is false in every scenario where the original is true and true in every scenario where the original is false.

What is the negation of the statement "every student attending this Zoom meeting is wearing red"?

Negate the following statements.

1. Every math student at UofT has a cellphone.
2. There is a country in the European Union with fewer than 1000 inhabitants.
3. I like math and physics.
4. Everyone in this class likes math and physics .

Negation, a harder example

Negation example

Negate "Every page in this book contains at least one word whose first and last letters both come alphabetically before M".

Hint: Try re-writing this sentence with a clause for each quantifier. For example, re-write this sentence starting with "For every page in this book, ... ". After you do this, negate systematically.

Mother

Let

$$H = \{Humans\}$$

True or False?

1. $\forall x \in H, \exists y \in H$ such that y gave birth to x.
2. $\exists y \in H$ such that $\forall x \in H$, y gave birth to x.

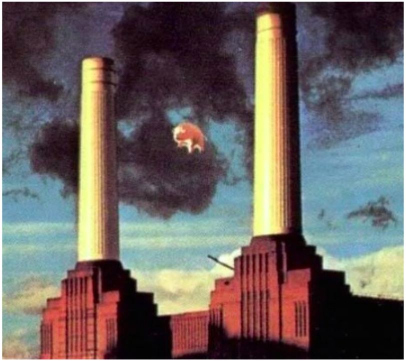
Order matters!

True or false:

1. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$
2. $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$

True or false?

1. All pigs in my room can fly.
2. There is a pig in my room that can fly.



Indecisive function

Construct a function f that satisfies all of the following properties at once:

- The domain of f is $\mathbb{R}$.
- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that

$$x < y \text{ and } f(x) < f(y)$$

- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that

$$x < y \text{ and } f(x) > f(y)$$