Announcements

- **Topics:** Applied optimization, Indeterminate forms, L'Hopital's rule
- **Homework:** Watch videos 6.13 - 6.18.

Maggie's farm

You are making a fence on Maggie's farm. Maggie has 300m of fencing and needs to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle. What is the largest area that the field can have?

Distance

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Fire

You hear a scream. You turn around and you see that I am on fire. Literally.

Luckily, you are next to a river.

I am 10 meters away from the river and you are 5 meters away from the point P on the river closest to me. You are carrying an empty bucket. You can run twice as fast with an empty bucket as you can run with a full bucket. How far from the point P should you fill your bucket in order to get to me with a bucket full of water as fast as possible?

Warm-up: What's the difference?

Which of the following is in indeterminate form?

1. $\lim_{x \to \infty} [x - x]$.
2. $\lim_{x \to \infty} x - \lim_{x \to \infty} x$.

What's the difference?

Indeterminate?

Which of the following are indeterminate forms for limits?
If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g such that

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This is how you show that $\frac{0}{0}$ is an indeterminate form.

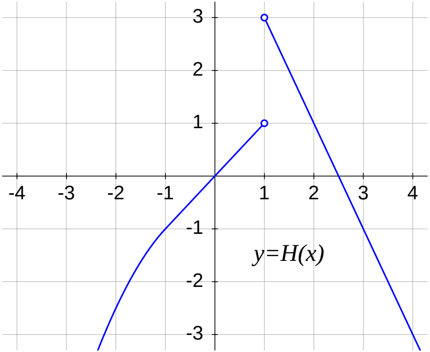
2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

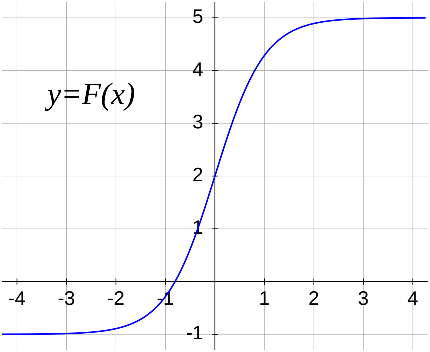
3. Prove that 1^{∞}, 0^0, and ∞^0 are indeterminate forms.
   (You will only get $c \geq 0$ this time)

Limits from graphs

Compute:

1. $\lim_{x \to 0} \dfrac{H(x)}{H(2+3x)-1}$
2. $\lim_{x \to 2} \dfrac{F^{-1}(x)}{x-2}$





Polynomial vs Exponential

Use L'Hôpital Rule to compute

$$\lim_{x\to\infty} \frac{x^7 + 5x^3 + 2}{e^x}.$$

Can you generalize this?

What's wrong with the following computation?

Computer $\lim_{x\to\infty} \frac{x+\sin(x)}{x}$.

Solution: $\lim_{x\to\infty} \frac{x+\sin(x)}{x} = \lim_{x\to\infty} \frac{1+\cos(x)}{1}$ by LH.

Therefore, $\lim_{x\to\infty} \frac{x+\sin(x)}{x}$ DNE since $1+\cos(x)$ oscillates between 0 and 2 as $x \to \infty$.

What does $\lim_{x\to\infty} \frac{x+\sin(x)}{x}$ actually equal?

Backwards L'Hôpital

Construct a polynomial P such that

$$\lim_{x \to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}.$$

Computations

Calculate:

1. $\displaystyle\lim_{x\to 2} \frac{x^2+2x-6}{x^2+3x-10}$
2. $\displaystyle\lim_{x\to\infty} x^3 e^{-x}$
3. $\displaystyle\lim_{x\to 0} x \sin\frac{2}{x}$
4. $\displaystyle\lim_{x\to\infty} x \sin\frac{2}{x}$
5. $\displaystyle\lim_{x\to\infty} x \cos\frac{2}{x}$
6. $\displaystyle\lim_{x\to 0} \frac{e^{2x^2}-\cos x}{x\sin x}$
7. $\displaystyle\lim_{x\to\infty} \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$
8. $\displaystyle\lim_{x\to 1} \left[(\ln x)\tan\frac{\pi x}{2}\right]$

Infinity minus infinity

Calculate:

1. $$\lim_{x \to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x} \right]$$
2. $$\lim_{x \to \infty} [\ln(x+2) - \ln(3x+4)]$$
3. $$\lim_{x \to 1} \left[\frac{2}{x^2 - 1} - \frac{1}{x-1} \right]$$
4. $$\lim_{x \to -\infty} \left[\sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right]$$

Exponential indeterminate forms

Calculate:

1. $\displaystyle\lim_{x\to\infty}\left(\frac{x+2}{x-2}\right)^{3x}$

2. $\displaystyle\lim_{x\to 0}\left[1+2\sin(3x)\right]^{4\cot(5x)}$

3. $\displaystyle\lim_{x\to 0^+} x^x$

4. $\displaystyle\lim_{x\to \frac{\pi}{2}^-}(\tan x)^{\cos x}$

5. $\displaystyle\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$