Announcements

- **Topics:** MVT, related rates
- **Homework:** Watch videos 6.3 - 6.12.

How many zeroes?

Let

$$f(x) = e^x - \sin x + x^2 + 10x$$

How many zeroes does f have? **Hint:** Differentiate. Is it obvious how many zeroes the derivative has? If not, differentiate again.

Zeroes of a polynomial

You probably learned in high school that a polynomial of degree n has at most n real zeroes. Now you can prove it!

Hint: Use induction.

MVT – True or False?

True or False

Consider $f(x) = |x|$ on the interval $[-\frac{1}{2}, 2]$.
There exists c in $(-\frac{1}{2}, 2)$ such that

$$f'(c) = \frac{f(2) - f(-\frac{1}{2})}{2 - (-\frac{1}{2})}$$

Proving difficult identities

Prove that, for every $x \geq 0$,

$$2 \arctan \sqrt{x} - \arcsin \frac{x-1}{x+1} = \frac{\pi}{2}$$

Positive derivative implies increasing

Use the MVT to prove

Theorem

Let $a < b$. Let f be a differentiable function on (a, b).

- IF $\forall x \in (a, b), f'(x) > 0$,
- THEN f is increasing on (a, b).

1. Recall the definition of what you are trying to prove.
2. **From that definition, figure out the structure of the proof.**
3. If you have used a theorem, did you verify the hypotheses?
4. Are there words in your proof, or just equations?

What is wrong with this proof?

Theorem

Let $a < b$. Let f be a differentiable function on (a, b).

- IF $\forall x \in (a, b), f'(x) > 0$,
- THEN f is increasing on (a, b).

Proof.

- From the MVT, $f'(c) = \dfrac{f(b) - f(a)}{b - a}$
- We know $b - a > 0$ and $f'(c) > 0$
- Therefore $f(b) - f(a) > 0$. Thus $f(b) > f(a)$.
- f is increasing.

□

True or False – Monotonicity and local extrema

Let I be an interval. Let f be a function defined on I. Let $c \in I$. Which implications are true?

1. IF f is increasing on I, THEN $\forall x \in I,\ f'(x) > 0$.
2. IF $\forall x \in I,\ f'(x) > 0$, THEN f is increasing on I.

3. IF f has a local extremum at c, THEN $f'(c) = 0$.
4. IF $f'(c) = 0$, THEN f has a local extremum at c.

5. IF f has local extremum at c, THEN f has an extremum at c
6. IF f has an extremum at c, THEN f has local extremum at c

Intervals of monotonicity

Let $g(x) = x^3(x^2 - 4)^{1/3}$.

Find the largest intervals on which this function is increasing or decreasing.

To save time, here is the first derivative:

$$g'(x) = \frac{x^2(11x^2 - 36)}{3(x^2 - 4)^{2/3}}$$

Fractional exponents

Let $g(x) = x^{2/3}(x-1)^3$.

Find local and global extrema of g on $[-1, 2]$.

Trig extrema

Let $f(x) = \dfrac{\sin x}{3 + \cos x}$.

Find the maximum and minimum values of f.

Inequalities

Prove that, for every $x \in \mathbb{R}$

$$e^x \geq 1 + x$$

Hint: Where is the function $f(x) = e^x - 1 - x$ increasing or decreasing? What is its minimum?

Related rates

Typical related rates problems: If there is a relationship between quantities and you know how one quantity is changing (usually with respect to time), then how does the other quantity change?

Example: You are filling up a perfectly spherical balloon. You inflate it at a rate of 1000 cm^3/s. At what rate is the radius of the balloon changing when the radius of the balloon is 20cm?

The formula for the volume of a sphere is $V = \frac{4\pi r^3}{3}$.

Related rates

A 10-meter long ladder is leaning against a vertical wall and sliding. The top end of the ladder is 8 meters high and sliding down at a rate of 1 meter per second. At what rate is the bottom end sliding away from the wall?

Math party

The MAT137 TAs wanted to rent a disco ball for their upcoming party. However, since they are poor, they could only afford a flashlight. At the party, one TA is designated the "human disco ball". The TA stands in the center of the room pointing the flashlight horizontally and spins at 3 revolutions per second. (Yes, they are that fast. Ask your TA to demonstratel if you don't believe me!) The room is square with side length 8 meters. At which speed is the light from the flashlight moving across the wall when it is 3 meters away from a corner?