Announcements

- **Topics:** Implicit differentiation, Derivatives of exponentials and logarithms, Related rates, Inverse functions
- **Homework:** Watch videos 4.3 - 4.8, 5.1 - 5.4

Implicit Differentiation

Function: For each input there is a unique output

Relation: A relationship between several variables with no well-defined idea of an input and an output, in particular no "uniqueness" of output.

Example: $x^2 + y^2 = 1$ is a relation but not a function.

We can still graph the relation by drawing the curve(s) of all (x, y) satisfying the equation and talk about the "tangent slope" at a given point on the graph.

However, saying something like find $\left.\frac{dy}{dx}\right|_{x=0}$ (usually) doesn't make sense.

Why?

Implicit differentiation

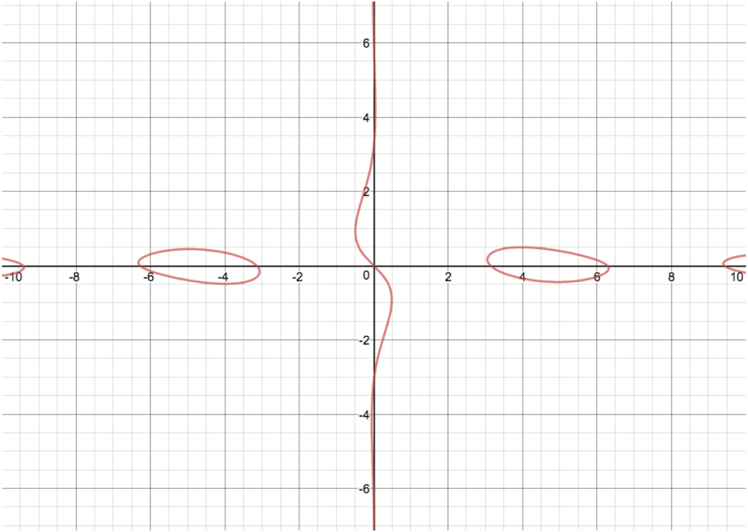
The equation

$$\sin(x + y) + xy^2 = 0$$

defines a function $y = h(x)$ near $(0, 0)$.

Compute:

1. $h(0)$
2. $h'(0) = \left.\frac{dy}{dx}\right|_{x=0,y=0}$
3. $h''(0) = \left.\frac{d^2y}{dx^2}\right|_{x=0,y=0}$
4. $h'''(0) = \left.\frac{d^3y}{dx^3}\right|_{x=0,y=0}$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Implicit differentiation

1. What is $\left.\frac{dx}{dy}\right|_{x=0,y=0}$? Make a guess from your previous work and check it by implicit differentiation.

2. What is $\left.\frac{d^2x}{dy^2}\right|_{x=0,y=0}$?

Warm up

Compute the derivative of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln x$
2. $f(x) = \pi^{\tan x}$
3. $f(x) = \ln\left[e^x + \ln\ln\ln x\right]$

Reminder: We know:

- $\dfrac{d}{dx} e^x = e^x$
- $\dfrac{d}{dx} a^x = a^x \ln a$
- $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$

Multiple choice

The derivative of x^x is:

a. $x(x^{x-1})$

b. $(ln(x)+1)x^x$

b. $ln(x)x^x$

Logrithmic differentiation

Find $\frac{dy}{dx}$:

1. $y = x^{x^x} + 1$
2. $x^y = x^2 + y^x$

Typical related rates problems: If there is a relationship between quantities and you know how one quantity is changing (usually with respect to time), then how does the other quantity change?

Example: You are filling up a perfectly spherical balloon. You inflate it at a rate of 1000 cm^3/s. At what rate is the radius of the balloon changing when the radius of the balloon is 20cm?

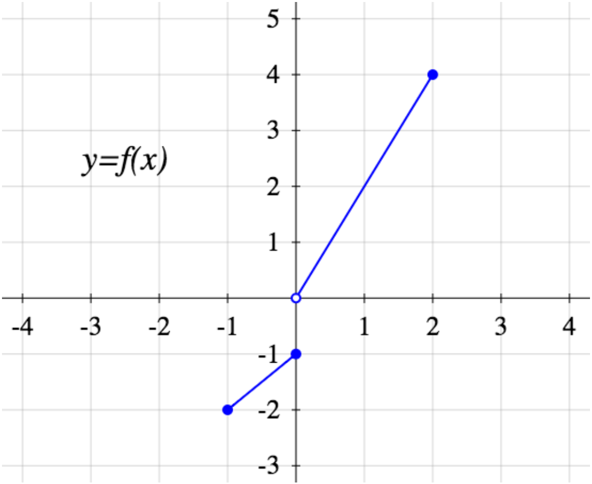
The formula for the volume of a sphere is $V = \frac{4\pi r^3}{3}$.

A 10-meter long ladder is leaning against a vertical wall and sliding. The top end of the ladder is 8 meters high and sliding down at a rate of 1 meter per second. At what rate is the bottom end sliding away from the wall?

Sleepy ants

Two ants are taking a nap. The first one is resting at the tip of the minute hand of a cuckoo clock, which is 25 cm long. The second one is resting at the tip of the hour hand, which is half the length. At what rate is the distance between the two ants changing at 3:30?

Inverse function from a graph



Calculate:

1. $f(2)$
2. $f(0)$
3. $f^{-1}(2)$
4. $f^{-1}(0)$
5. $f^{-1}(-1)$

Absolute value and inverses

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Find an equation for $h^{-1}(x)$.
3. Sketch the graphs of h and h^{-1}.
4. Verify that for every $x \in \mathbb{R} =$ range of $h =$ domain of h^{-1}, $h(h^{-1}(x)) = x$, and that
   for every $x\mathbb{R} =$ domain of $h =$ range of h^{-1}, $h^{-1}(h(x)) = x$.

An interesting example

Let $f(x) = x^2 \sin \dfrac{1}{x}$.

1. Calculate $f'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $f'(0)$.
3. Is f continuous at 0?
4. Is f differentiable at 0?
5. Is f' continuous at 0?