Announcements

- **Topics:** IVT, EVT, Definition of derivatives, differentiation rules
- **Homework:** Watch videos 3.6, 3.7, 3.9 -3.12

Computations using limit laws

Given a function g s.t.

$$\lim_{x \to 0} \frac{g(x)}{x^2} = 2.$$

Use it to compute the following limits (or explain that they don't exist).

1. $\lim_{x \to 0} \frac{g(x)}{x}$
2. $\lim_{x \to 0} \frac{g(x)}{x^4}$
3. $\lim_{x \to 0} \frac{g(3x)}{x^2}$

Computations

Compute:

1. $\lim_{x \to 2} \frac{|x^2-4|}{x^2-5x+6}$ Hint: Calculate left/right limits to get rid of absolute value sign.
2. $\lim_{x \to 4} \frac{x^2-5x+4}{\sqrt{x}-2}$ Hint: Try multiplying and dividing by the conjugate of the denominator.
3. $\lim_{x \to \infty} \frac{x^3+\sqrt{2x^6+1}}{2x^3+\sqrt{x^5+1}}$ Hint: Try factoring out the dominant term from the numerator and the denominator.
4. $\lim_{x \to -\infty} x - \sqrt{x^2+x}$ Hint: You can tell what this goes to by looking at the two limits separately.
5. $\lim_{x \to -\infty} x + \sqrt{x^2+x}$ Hint: Try multiplying and dividing by the conjugate.

Definition of maximum

Let f be a function with domain I.
Which one (or ones) of the following is (or are) a definition of
"f has a maximum on I"?

1. $\forall x \in I, \exists C \in \mathbb{R}$ s.t. $f(x) \leq C$
2. $\exists C \in I$ s.t. $\forall x \in I, f(x) \leq C$
3. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) \leq C$
4. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) < C$

More on the definition of maximum

Let f be a function with domain I.

What does each of the following mean?

1. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I$, $f(x) \leq C$
2. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I$, $f(x) < C$
3. $\exists a \in I$ s.t. $\forall x \in I$, $f(x) \leq f(a)$
4. $\exists a \in I$ s.t. $\forall x \in I$, $f(x) < f(a)$

EVT is best possible?

Recall the statement of EVT.

Find/draw a continuous function f which is continuous on $[0, 1)$ which doesn't have a maximum.

Find/draw a continuous function f which is continuous on $[0, 1)$ which has neither a maximum nor a minimum.

Can this be proven? (Use IVT)

1. Prove that at some point in your life your height was exactly 1m.
2. Prove that there exists a time of the day when the hour hand and the minute hand of a clock form an angle of exactly 23 degrees.
3. During a Raptors basketball game, at half time the Raptors have 51 points. Prove that at some point they had exactly 26 points.

Existence

Prove that the equation

$$x^4 - 2x = 100$$

has at least two solutions.

A quick tangent line

What is the equation of the line tangent to the graph of $y = x$ at the point with x–coordinate 7?

1. $y = x + 7$
2. $y = x$
3. $y = 7$
4. $x = 7$
5. There is no tangent line at that point.
6. There is more than one tangent line at that point.

Absolute value and tangent lines

At (0,0) the graph of $y = |x|$...

1. ... has one tangent line: $y = 0$
2. ... has one tangent line: $x = 0$
3. ... has two tangent lines $y = x$ and $y = -x$
4. ... has no tangent line

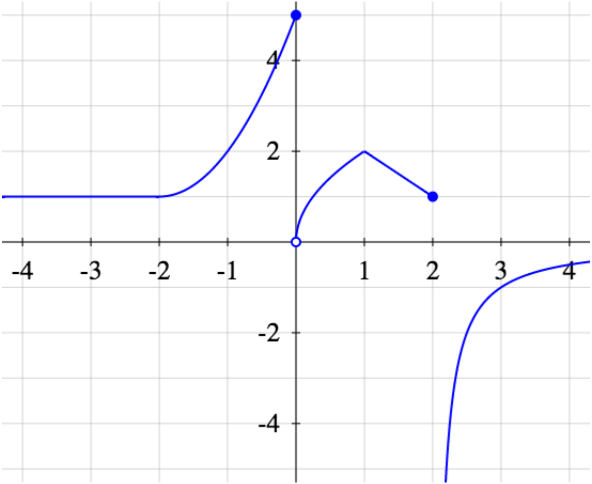
Absolute value and derivatives

Let $h(x) = x|x|$. What is $h'(0)$?

1. It is 0.
2. It does not exist because $|x|$ is not differentiable at 0.
3. It does not exist because the right- and left-limits, when computing the derivative, are different.
4. It does not exist because it has a corner.

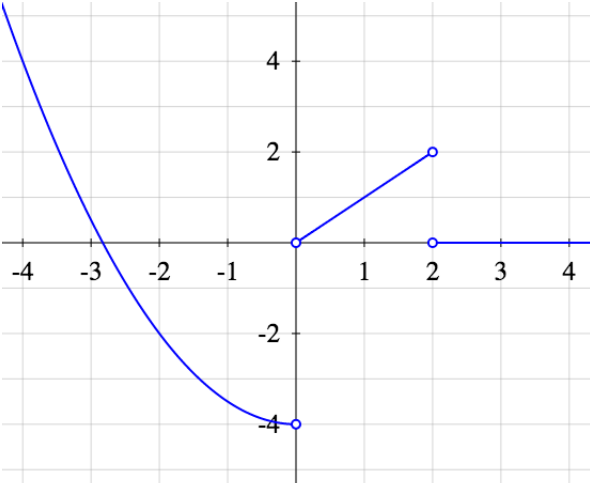
Intuitive idea of the derivative

Graph the derivative of this function.



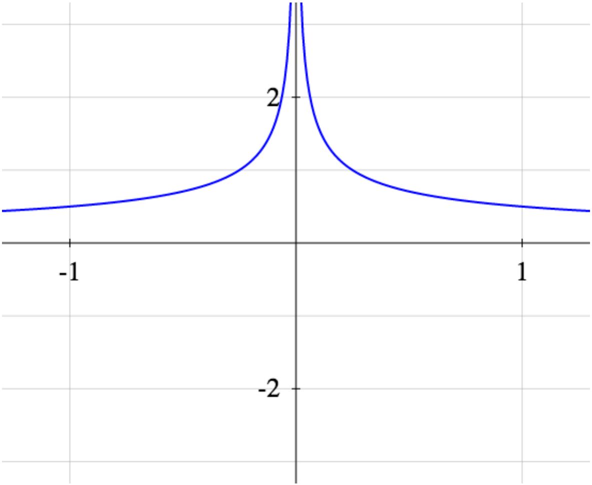
Intuitive idea of the derivative

Below is the graph of the derivative of some function f. We know f is continuous and $f(0) = 0$. Graph f.



Intuitive idea of the derivative

Below is the graph of the derivative of some function f. We know f is continuous and $f(0) = 0$. Graph f.



Derivatives from the definition

Let

$$g(x) = \frac{2}{\sqrt{x}}$$

Calculate $g'(4)$ directly from the definition of derivative as a limit.

Estimations

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: Consider the values you know for $f(x) = \sqrt[20]{x}$ and its derivative.

Estimations

Without using a calculator, estimate $\sqrt[20]{1.01}$ as well as you can.

Hint: Consider the values you know for $f(x) = \sqrt[20]{x}$ and its derivative.

Product of 3 functions

Given f_1, f_2 and f_3 differentiable on $\mathbb{R}$, what can you say $(f_1(x)f_2(x)f_3(x))' = ?$

Let $g(x) = \dfrac{1}{x^3}$.

Calculate the first few derivatives.
Make a conjecture for a formula for the n-th derivative $g^{(n)}(x)$.
Prove it.

Computations

Compute the derivative of the following functions:

1. $$f(x) = x^{100} + 3x^{30} - 2x^{15}$$
2. $f(x) = \sqrt[3]{x} + 6$
3. $f(x) = \dfrac{4}{x^4}$
4. $f(x) = \sqrt{x}\,(1 + 2x)$
5. $f(x) = \dfrac{x^6 + 1}{x^3}$
6. $f(x) = \dfrac{x^2 - 2}{x^2 + 2}$