Announcements

- **Topics:** Absolute values, inequalities, definitions of limits
- **Homework:** Watch videos 2.7 - 2.13
- **PS 2** has been posted. It's due next week Wednesday.

Properties of inequalities

Given $a, b, c \in \mathbb{R}$.

Assume $a < b$. What can we (always) conclude?

1. $a + c < b + c$
2. $a - c < b - c$
3. $ac < bc$
4. $a^2 < b^2$
5. $\frac{1}{a} < \frac{1}{b}$

Properties of absolute value

Given $a, b \in \mathbb{R}$. What can we (always) conclude?

1. $|ab| = |a||b|$
2. $|a + b| = |a| + |b|$

Sets described by distance

Given $a \in \mathbb{R}$. Given $\delta > 0$.

What are the following sets? Describe them in terms of intervals.

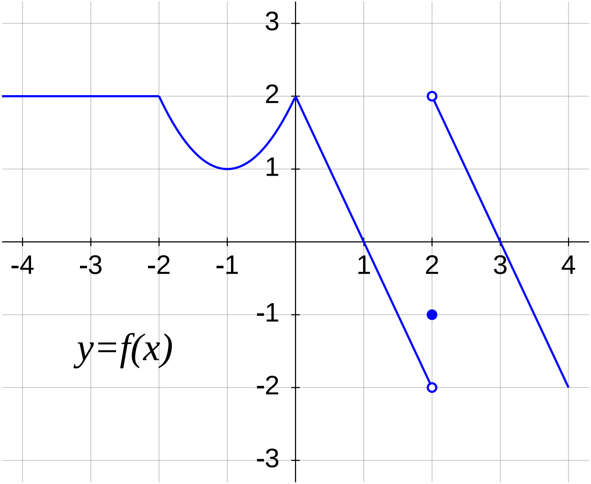
1. $A = \{x \in \mathbb{R} : |x| < \delta\}$
2. $B = \{x \in \mathbb{R} : |x| > \delta\}$
3. $C = \{x \in \mathbb{R} : |x - a| < \delta\}$
4. $D = \{x \in \mathbb{R} : 0 < |x - a| < \delta\}$

Implications

Find **all** positive values of A, B, and C which makes the following implications true.

1. $(\forall x \in \mathbb{R},)\ |x-3| < 1 \implies |2x-6| < A.$
2. $(\forall x \in \mathbb{R},)\ |x-3| < B \implies |2x-6| < 1.$
3. $(\forall x \in \mathbb{R},)\ |x-3| < 1 \implies |x-3.5| < C.$

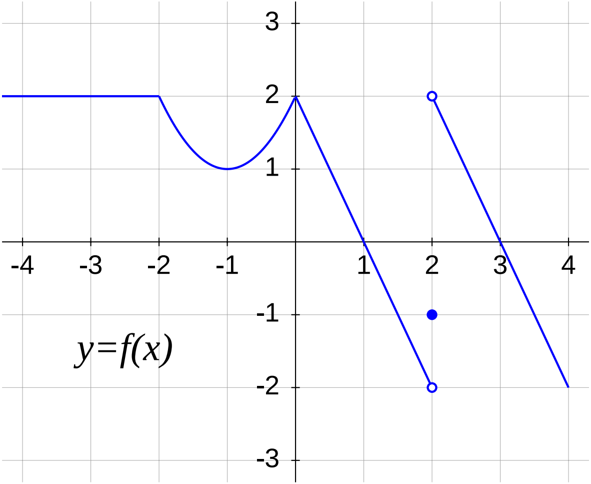
Limits from a graph



Do the following limits exist?

1. $\lim_{x \to 2} f(x)$
2. $\lim_{x \to 0} f(f(x))$

Limits from a graph



Find the value of

1. $\lim\limits_{x \to 2} f(x)$
2. $\lim\limits_{x \to 0} f(f(x))$
3. $\lim\limits_{x \to 2} [f(x)]^2$
4. $\lim\limits_{x \to -3} f(f(x))$

Floor

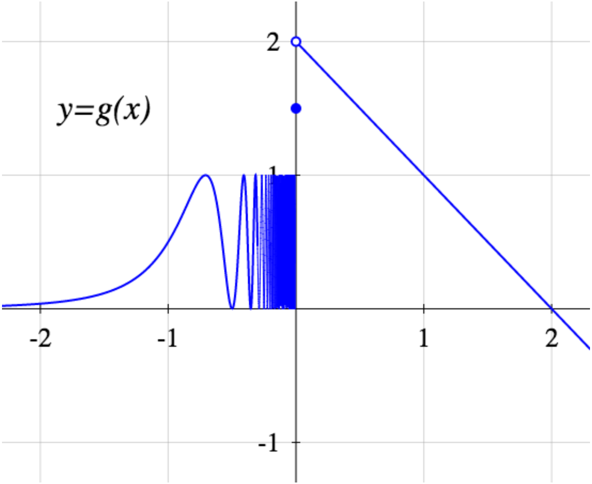
Given a real number x, we defined the *floor of* x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example:

$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$. Then compute:

1. $\lim\limits_{x \to 0^+} \lfloor x \rfloor$
2. $\lim\limits_{x \to 0^-} \lfloor x \rfloor$
3. $\lim\limits_{x \to 0} \lfloor x \rfloor$
4. $\lim\limits_{x \to 0} \lfloor x^2 \rfloor$

More limits from a graph



Find the value of

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g(\lfloor x \rfloor)$
4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \lfloor \frac{g(x)}{2} \rfloor$
7. $\lim_{x\to 0^-} g(\lfloor x \rfloor)$

Formal definition of a limit

Definition of a limit

Given a, $L \in \mathbb{R}$ and

f a function defined in an open interval around a, except possibly at a,

we say that $\lim_{x \to a} f(x) = L$ iff

$\forall \epsilon > 0$, $\exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$.

Translation

Translation of $\forall \epsilon > 0$, $\exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$.

$\forall \epsilon > 0$	"If you give me any distance ϵ..."
$\exists \delta > 0$ s.t.	"... I can find a distance δ such that..."
$0 < \|x - a\| < \delta \implies$	"... if x is within δ of (but not equal to) a..."
$\|f(x) - L\| < \epsilon$.	"... then $f(x)$ is within ϵ of L."

Formal definition of a one-sided limit

Given a ,$L \in \mathbb{R}$.

Write down the definition of $\lim_{x \to a^+} f(x) = L$.

Exercise: Write down the definition of $\lim_{x \to a^-} f(x) = L$.

Existence of limits

Given $a \in \mathbb{R}$.

Write down the definition of the following statments:

1. $\lim_{x \to a} f(x)$ exists.
2. $\lim_{x \to a} f(x)$ does not exist.

Existence of limits

Given $a, L \in \mathbb{R}$.

Write down the definition of the following statments:

1. $\lim_{x \to a} f(x) = \infty$.
2. $\lim_{x \to \infty} f(x) = L$.

Hint: For 1, you want to replace the two parts with ϵ in the $\epsilon - \delta$ definition for limits. Instead of saying $f(x)$ gets arbitrarily close to L as x gets close to a, you want to say $f(x)$ gets arbitrarily large. How can you do this?