Announcements

- **Topics:** Negation, implications, simple proofs, inductive proofs
- **Homework:** Watch videos 2.1 - 2.6.

Homework: Negation, a harder example

Negation example

Negate "Every page in this book contains at least one word whose first and last letters both come alphabetically before M".

Hint: Try re-writing this sentence with a clause for each quantifier. For example, re-write this sentence starting with "For every page in this book, ... ". After you do this, negate systematically.

Mother

Let

$$H = \{Humans\}$$

True or False?

1. $\forall x \in H, \exists y \in H$ such that y gave birth to x.
2. $\exists y \in H$ such that $\forall x \in H$, y gave birth to x.

Order matters!

True or false:

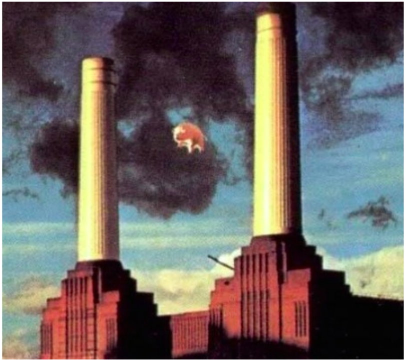
1. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$
2. $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$

Prove it!

Vacuous truth: Pigs on the wing

True or false?

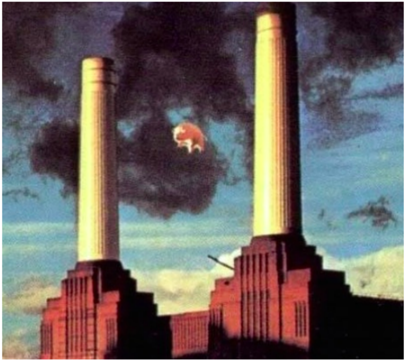
1. All pigs in my room can fly.
2. There is a pig in my room that can fly.



Vacuous truth: Testing your understanding

True or false?

1. $\forall x \in \emptyset, \exists y \in \emptyset$ s.t. some statement about x and y.
2. $\exists x \in \emptyset$ s.t. $\forall y \in \emptyset$, some statement about x and y.



Am I lying?

I tell you: "If you get 80% or more on your midterm, then I will give you a piece of chocolate."

In which of the following scenarios would I have lied (i.e. said something false)?

1. You get 80% on your test, and I give you a piece of chocolate.
2. You get 70% on your test, and I don't give you a piece of chocolate.
3. You get 100% on your test, and I don't give you a piece of chocolate.
4. You get 60% on your test, and I give you a piece of chocolate.
5. I give everybody a piece of chocolate.
6. You get 60% on your test.

The Wason selection task

Every card on the table has a number on one side and a letter on the other side.

I tell you: "(For all the cards on the table.) If a card has a vowel on one side then it must have an even number on the other side."

You see 4 cards with "B", "7", "8", "A".

Which cards do you have to turn over to make sure I'm telling the truth?

The Wason selection task negation

What is the negation of the statement

"(For all the cards on the table.) If a card has a vowel on one side then it must have an even number on the other side"?

1. "If a card has a vowel on one side then it must have an odd number on the other side."
2. "If a card has a consonant on one side then it must have an even number on the other side."
3. "If a card has a consonant on one side then it must have an odd number on the other side."
4. "There is a card with a vowel on one side and an odd number on the other side."

Mathier conditionals

True or false?

1. $\forall x \in \mathbb{R},\ x > 0 \implies x \geq 0.$
2. $\forall x \in \mathbb{R},\ x \geq 0 \implies x > 0.$

Definition

Even and Odd

For $x \in \mathbb{R}$, give a mathematical definition for the statements "x is an even number". Do the same for the statement "x is an odd number".

Definition

Let $x \in \mathbb{R}$.

x is even $\iff$???

x is odd $\iff$???

A claim about odd and even numbers

Claim

The sum of two odd numbers is even.

This should be interpreted as "The sum of any two odd numbers is even.

Claim

$\forall x, y \in \mathbb{R}$, x, y are odd $\implies$ $x + y$ is even.

Bad proof

Claim

The sum of any two odd numbers is even.

Proof

1 is odd.

3 is odd.

$1 + 3 = 4$ is even.

Bad proof

Claim

The sum of any two odd numbers is even.

Proof

For all n:

EVEN $+$ EVEN $=$ EVEN

EVEN $+$ ODD $=$ ODD

ODD $+$ ODD $=$ EVEN

Bad proof

Claim

The sum of any two odd numbers is even.

Proof

$(2a+1)+(2b+1)=2a+2b+2$ is even.

Proof exercise

Exercise

Prove: The sum of any two odd numbers is even (i.e. $\forall x, y \in \mathbb{R}$, x, y are odd $\implies$ $x + y$ is even).

Proof exercise

Exercise

Prove: The sum of any two odd numbers is even (i.e. $\forall x, y \in \mathbb{R}$, x, y are odd $\implies x + y$ is even).

Standard induction

Standard induction

We have statements S_n dependent on natural numbers n. To prove $\forall n \in \mathbb{N}$, S_n is true using standard induction we need to prove the following two statements:

1. ???
2. ???

For example, S_n can be the statement "$0 + 1 + 2 + ... + n = \frac{(n)(n+1)}{2}$".

What if?

If you managed to show the following instead, what can you prove?

Non-standard induction 1

1. S_3 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+1}$" is true.

Non-standard induction 2

1. S_1 is true.
2. "$\forall n > 1,\ S_n \implies S_{n+1}$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+3}$" is true.

Nonstandard induction 4

1. S_7 is true.
2. "$\forall n > 3,\ S_{n+1} \implies S_n$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+3}$" is true.

What else do I need to prove to show S_n is true for all positive integers n?

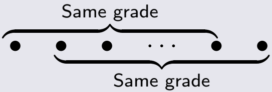
All 0s?

Theorem

$\forall N \in \mathbb{N}$, *in every set of N MAT137 students, those students will get the same grade.*

Proof.

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for N. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ students. By induction hypothesis:
  - The first N students have the same grade.
  - The last N students have the same grade.



Hence the $N + 1$ students all have the same grade as well.

$\square$