Announcements

- **Topics:** Optimization, Indeterminate forms, L'Hopital's Rule
- **Homework:** Watch videos 6.11 - 6.16.

Proving difficult identities

Prove that, for every $x \geq 0$,

$$\arcsin \frac{1-x}{1+x} + 2 \arctan \sqrt{x} = \frac{\pi}{2}$$

Hint: Take derivatives.

Warm-up: What's the difference?

Which of the following is in indeterminate form?

1. $\lim_{x \to \infty} [x - x]$.
2. $\lim_{x \to \infty} x - \lim_{x \to \infty} x$.

What's the difference?

Indeterminate?

Which of the following are indeterminate forms for limits?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This is how you show that $\dfrac{0}{0}$ is an indeterminate form.

2. Show the same way that $\dfrac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.
3. Homework: show that 1^∞, 0^0, and ∞^0 are indeterminate forms. (You will not be able to get all $c \in \mathbb{R}$ this time.)

What's wrong with the following computation?

Since $\lim\limits_{x \to \infty} \frac{x+\sin(x)}{x}$ is in indeterminate form,

$\lim\limits_{x \to \infty} \frac{x+\sin(x)}{x} = \lim\limits_{x \to \infty} \frac{1+\cos(x)}{1}$ by LH.

Therefore, $\lim\limits_{x \to \infty} \frac{x+\sin(x)}{x}$ DNE since $1 + \cos(x)$ oscillates between 0 and 2 as $x \to \infty$.

What does $\lim\limits_{x \to \infty} \frac{x+\sin(x)}{x}$ actually equal?

Infinity minus infinity

Compute:

1. $\displaystyle\lim_{x\to\infty} [\ln(x+2) - \ln(3x+4)]$

2. $\displaystyle\lim_{x\to-\infty} \left[\sqrt{x^2+3x} - \sqrt{x^2-3x}\right]$

3. $\displaystyle\lim_{x\to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x}\right]$

4. $\displaystyle\lim_{x\to 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right]$

Exponential indeterminate forms

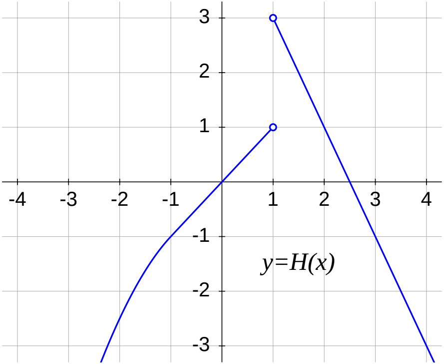
Compute:

1. $$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$
2. $$\lim_{x\to\frac{\pi}{2}^{-}}(\tan x)^{\cos x}$$
3. $$\lim_{x\to 0}\left[1+2\sin(3x)\right]^{4\cot(5x)}$$
4. $$\lim_{x\to\infty}\left(\frac{x+2}{x-2}\right)^{3x}$$
5. $$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$$

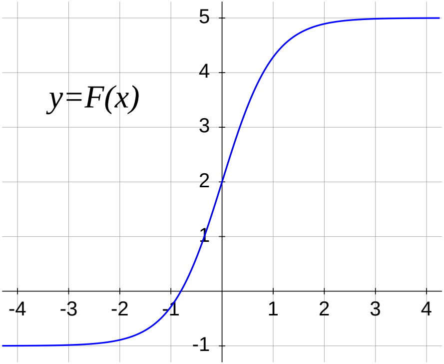
Limits from graphs

Compute:

1. $$\lim_{x \to 0} \frac{H(x)}{H(2+3x)-1}$$



2. $$\lim_{x \to 2} \frac{F^{-1}(x)}{x-2}$$



Backwards L'Hôpital

1. Construct a polynomial P such that

$$\lim_{x \to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}$$

2. Find $a \in \mathbb{R}$ and $n \in \mathbb{N}$ such that the limit

$$\lim_{x \to 0} \frac{\sin x - ax^n}{x^3}$$

exists. What is the value of the limit?

Maggie's farm

Maggie has 300m of fencing and needs to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle. What is the largest area that the field can have?

Distance

Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

Fire

You hear a scream. You turn around and you see that Qin is on fire. Literally.

At first, you think maybe you should just let Qin burn - perhaps they'll cancel the test if that happens. After a moment, your pesky conscience sets in.

Luckily, you are next to a river.

Qin is 10 meters away from the river and you are 5 meters away from the point P on the river closest to Qin. You are carrying an empty bucket. You can run twice as fast with an empty bucket as you can run with a full bucket. How far from the point P should you fill your bucket in order to get to Qin with a bucket full of water as fast as possible?