MAT 137
Tutorial #9– L'Hôpital's Rule
July 3-4, 2019

1. Compute the following limits:

   (a) $\lim_{t\to 0} \frac{\tan(3t)}{\ln(1+2t)}$

   (b) $\lim_{h\to 2} \frac{h^3 - 5h^2 + 3h + 6}{h^3 - h^2 - 3h + 2}$

   (c) $\lim_{t\to 0} \frac{1-\cos(3t)}{t\ln(1+t)}$

   (d) $\lim_{x\to\infty} \left(x^7 - 5x^3 + 2\right) e^{-x}$

   (e) $\lim_{x\to 1} \frac{(x-1)\sin x}{e^x \cos x}$

   (f) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

   (g) $\lim_{x\to\infty} x \tan\frac{3}{x}$

   (h) $\lim_{x\to 0} \left(\frac{1}{x\sin x} - \frac{1}{x\tan x}\right)$

   (i) $\lim_{x\to\infty} \frac{\sqrt{x^4+5x^3}-x^2}{x}$

   (j) $\lim_{x\to\infty} \frac{\sin x}{x}$

2. Compute the following limits:

   (a) $\lim_{x\to 0} (\cos x)^{1/x^2}$

   (b) $\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$

   (c) $\lim_{x\to 0^+} x^{\sqrt{x}}$

   (d) $\lim_{x\to 1} (2-x)^{\tan(\pi x/2)}$

A historical question

3. The first appearance in print of L'Hôpital's Rule was in the book *Analyse des Infiniment Petits* published by the Marquis de l'Hôpital in 1696. This was the first calculus *textbook* ever published and the example that the Marquis used in that book to illustrate his rule was to find the limit of the function

$$y = \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{aax}}{a - \sqrt[4]{ax^3}}$$

   as x approaches a, where $a > 0$ is a constant. (At that time it was common to write aa instead of a^2.) Solve this problem.[1]

[1] Problem taken from *Stewart, James: Calculus Early Trascendental*, 7th Ed.