Today's topics and news

- Topic: Integral test, comparison tests (BCT and LCT), alternating series
- **Homework for Friday:** Watch videos 13.18, 13.19, and 14.1 - 14.4

Rapid questions: review of improper integrals

1. $\int_1^\infty \frac{1}{x^2}\,dx$
2. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$
4. $\int_0^1 \frac{1}{x^2}\,dx$
5. $\int_0^1 \frac{1}{\sqrt{x}}\,dx$
6. $\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\int_0^\infty \frac{1}{x^2}\,dx$
8. $\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF $\lim_{n\to\infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
6. IF $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
7. IF $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n \right] = 0$.
8. IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.

Problem

Suppose $\sum_{n=0}^{\infty} a_n$ converges and $\forall n \in \mathbb{N}, a_n \neq 0$, what can you say about $\sum_{n=0}^{\infty} \frac{1}{a_n}$?

List of tests

We have learned:

- Divergence test (WARNING: This can only tell you if a series diverges. It will never check if a series converges.)

Today we will talk about:

- Integral test
- BCT
- LCT
- Alternating series test
- Absolute convergence test

Rapid questions: For which values of $p \in \mathbb{R}$ are these series convergent? What does the series converge to?

1. $$\sum_{n=1}^{\infty} \frac{1}{p^n}$$
2. $$\sum_{n=1}^{\infty} \frac{1}{n^p}$$
3. $$\sum_{n=1}^{\infty} p^n$$
4. $$\sum_{n=1}^{\infty} n^p$$

More rapid questions: Convergent or divergent?

1. $$\sum_{n}^{\infty} \frac{n^{10} + 17n^7 + 3}{n^{11}}$$
2. $$\sum_{n}^{\infty} \frac{\sqrt[3]{n^2+1}+1}{\sqrt{n^4+n}+n+1}$$

Slower questions: convergent or divergent?

Using IT, BCT, or LCT, check if the following converges or diverges:

1. $\displaystyle\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$
2. $\displaystyle\sum_{n}^{\infty} \frac{1}{n \ln n}$
3. $\displaystyle\sum_{n}^{\infty} \sin \frac{1}{n^2}$
4. $\displaystyle\sum_{n}^{\infty} \frac{1}{n(\ln n)^3}$
5. $\displaystyle\sum_{n}^{\infty} \frac{(\ln n)^{20}}{n^2}$
6. $\displaystyle\sum_{n}^{\infty} e^{-n^2}$

New series based on a convergent series

We know

- $\forall n \in \mathbb{N}$, $0 < a_n < 1$.
- the series $\sum_{n}^{\infty} a_n$ is convergent

Determine whether the following series are convergent, divergent, or we do not have enough information to decide:

1. $\sum_{n}^{\infty} \sin a_n$
2. $\sum_{n}^{\infty} \cos a_n$
3. $\sum_{n}^{\infty} \sqrt{a_n}$

An AST example

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty}(-1)^n \frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

Estimation

Estimate the sum

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

with an error smaller than 0.001.

All the conditions of AST are necessary

Construct a series of the form $\sum_{n=1}^{\infty}(-1)^n b_n$ such that

- $b_n > 0$ for all $n \geq 1$
- $\lim_{n\to\infty} b_n = 0$
- the series $\sum_{n=1}^{\infty}(-1)^n b_n$ is divergent.

True or False – Absolute Values

17. IF $\{a_n\}_{n=1}^{\infty}$ is convergent, THEN $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent.
18. IF $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent, THEN $\{a_n\}_{n=1}^{\infty}$ is convergent.
19. IF $\sum_{n=1}^{\infty} a_n$ is convergent, THEN $\sum_{n=1}^{\infty} |a_n|$ is convergent.
20. IF $\sum_{n=1}^{\infty} |a_n|$ is convergent, THEN $\sum_{n=1}^{\infty} a_n$ is convergent.