Today's topics and news

- Topic: LCT, Series
- **Homework for Wednesday:** Watch videos 13.10 - 13.17.
- **Homework for Friday:** Watch videos 13.18, 13.19, and 14.1 - 14.4

Using LCT

Does the following improper integral converge or diverge?

$$\int_1^\infty \frac{1}{\sqrt{x-1}(x+2)}dx$$

Convergent or divergent?

1. $$\int_1^\infty \frac{x^3+2x+7}{x^5+11x^4+1}\,dx$$
2. $$\int_1^\infty \frac{1}{\sqrt{x^2+x+1}}\,dx$$
3. $$\int_0^1 \frac{3\cos x}{x+\sqrt{x}}\,dx$$
4. $$\int_0^1 \cot x\,dx$$
5. $$\int_0^1 \frac{\sin x}{x^{3/2}}\,dx$$
6. $$\int_0^\infty e^{-x^2}\,dx$$
7. $$\int_2^\infty \frac{(\ln x)^{10}}{x^2}\,dx$$
8. $$\int_0^1 \frac{\arcsin x}{x^{3/2}}\,dx$$

Hint: For some these, $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ is useful.

What is wrong with this calculation? Fix it

Claim:

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

"Justification"

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) \; - \; \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \ldots) \; - \; (\ln 3 + \ln 4 + \ldots) \\
&= \ln 2
\end{aligned}$$

Trig series: convergent or divergent?

1. $\sum_{n=0}^{\infty} \sin(n\pi)$
2. $\sum_{n=0}^{\infty} \cos(n\pi)$

Hint: Compute the first few partial sums.

A telescopic series

I want to calculate the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

1. Find a formula for the k-th partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^2+2n}$.

   Hint: Write $\frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

Challenge: Compute $S = \sum_{n=2}^{\infty} \frac{3-5n}{n^3-n}$.

Geometric series

Calculate the value of the following series:

1. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$
2. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \dots$
3. $\frac{3}{2} - \frac{9}{4} + \frac{27}{8} - \frac{81}{16} + \dots$
4. $1 + \frac{1}{2^{0.5}} + \frac{1}{2} + \frac{1}{2^{1.5}} + \frac{1}{2^2} + \frac{1}{2^{2.5}} + \dots$
5. $\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{2^{2n+1}}$
6. $\sum_{n=k}^{\infty} x^n$

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$
2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$
3. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.
4. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

6. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing, THEN $\forall n \geq 0$, $a_n > 0$.

7. IF $\lim_{n\to\infty} a_n = 0$, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

8. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.

True or False – The Necessary Condition

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^{\infty} a_n$ is convergent.
2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_n^{\infty} a_n$ is divergent.
3. IF $\sum_n^{\infty} a_n$ is convergent THEN $\lim_{n\to\infty} a_n = 0$.
4. IF $\sum_n^{\infty} a_n$ is divergent THEN $\lim_{n\to\infty} a_n \neq 0$.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF $\lim_{n\to\infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
6. IF $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
7. IF $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n \right] = 0$.
8. IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.

Problem

Suppose $\sum_{n=0}^{\infty} a_n$ converges and $\forall n \in \mathbb{N}, a_n \neq 0$, what can you say about $\sum_{n=0}^{\infty} \frac{1}{a_n}$?