Today's topics and news

- Topic: Integration by parts, integration of trig functions and rational functions
- **Homework for Wednesday:** Watch videos 10.1, 10.2, 11.1 and 11.2
- **Homework for Friday:** Watch videos 11.3 - 11.8.

Computation practice: Integration by parts

Use integration by parts (possibly in combination with other methods) to compute:

1. $\int xe^{-2x}\,dx$
2. $\int \ln x\,dx$
3. $\int x\arctan x\,dx$
4. $\int \sin\sqrt{x}\,dx$
5. $\int x^2\sin x\,dx$
6. $\int x^2\arcsin x\,dx$
7. $\int e^{\cos x}\sin^3 x\,dx$
8. $\int e^{ax}\sin(bx)dx$

A reduction formula

Prove the following formula:

$$\int \sin^n(x)dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x)+\frac{n-1}{n}\int \sin^{n-2}(x)dx.$$

Hint: Start with an integration by parts on $\int \sin^n(x)dx$.

This formula is sometimes useful for solving trignometric integrals. We will discuss another way of integrating $\sin^n(x)$ in a few slides.

Problem

Given a function $g(x)$ s.t. $g''(x)$ is continuous on $\mathbb{R}$ and that

$$\int_0^{2\pi} g(x)sin(x)dx + \int_0^{2\pi} g''(x)sin(x)dx = 2.$$

If $g(2\pi) = 1$, what is $g(0)$?

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

1. $\int \sin^{10} x \cos x \; dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int \cos^2 x \, dx$
4. $\int e^{\cos x} \cos x \sin^5 x \, dx$
5. $\int \sin^4 x \, dx$
6. $\int \sec x \, dx$ (hint: multiply and divide by $\sec x + \tan x$)

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

Hint: You will need

- $\dfrac{d}{dx}[\tan x] = \ldots$
- $\dfrac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

A pair of mysterious functions

Suppose functions $\alpha(x)$, $\beta(x)$ satisfy the following:

1. $\alpha'(x) = 2\beta(x)$
2. $\beta'(x) = \frac{1}{2}\alpha(x)$
3. $\alpha(x)^2 - \beta(x)^2 = 1$

Do not try to find formulas for $\alpha(x)$ and $\beta(x)$. Integrate the following:

1. $\int \sin(x)\alpha(x)dx$
2. $\int \frac{\beta(x)^3}{\alpha(x)^4}dx$

Rational integrals

1. Calculate $\int \frac{1}{x+a}\,dx$
2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x+3}$
3. Calculate $\int \frac{-x+6}{x^2+3x}\,dx$
4. Calculate $\int \frac{1}{x^2+3x}\,dx$
5. Calculate $\int \frac{1}{x^3-x}\,dx$

Repeated factors

1. Calculate $\int \frac{1}{(x+1)^n}\,dx$ for $n > 1$
2. Calculate $\int \frac{(x+1)-1}{(x+1)^2}\,dx$
3. Calculate $\int \frac{2x+6}{(x+1)^2}\,dx$
4. Calculate $\int \frac{x^2}{(x+1)^3}\,dx$
5. How would you calculate $\int \frac{\text{polynomial}}{(x+1)^3}\,dx$?

Irreducible quadratics

1. Calculate $\int \frac{1}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$.
   Hint: These two are very short.
2. Calculate $\int \frac{2x+3}{x^2+1}\,dx$
3. Calculate $\int \frac{x^3}{x^2+1}\,dx$
4. Calculate $\int \frac{x}{x^2+x+1}\,dx$

   Hint: Transform it into one like the previous ones

Messier rational functions

1. How could we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)}dx \ ?$$

2. How could we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)x^4(x^2+1)(x^2+4x+7)}dx \ ?$$