Today's topics and news

- Topics: Taylor Series applications

Evaluating series using Taylor series

Evaluate the following series:

1. $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$

2. $\sum_{n=2}^{\infty} \frac{(-2)^n}{(2n+1)!}$

3. $\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{(2n)!} 2^n$

4. $\sum_{n=0}^{\infty} \frac{2^n}{(2n)!}$

Limits

Evaluate the following limits using Taylor series.

1. $\lim_{x \to 0} \frac{x^2 e^x}{\cos(x) - 1}$
2. $\lim_{x \to 0} \frac{\cos(x) - 1 + \frac{1}{2} x \sin(x)}{(\ln(1+x))^4}$

Maclaurin series and derivatives at 0

Let f be an analytic function defined on some interval centred at 0. We define a new function g via the equation $g(x) = f(x^2)$. Finding $g^n(0)$ in terms of the derivatives of f at 0.

Hint: Write a Maclaurin series for g in two different ways.

Differential equations through Taylor series

We are looking for functions $y(t)$ which solves the simple harmonic oscillator differential equation:

$$y''(t) + y(t) = 0.$$

You can probably guess what the solution to this equation is already without dong any work. However, we can also solve, or at least approximate solutions to, differential equations using Taylor series.

Differential equations through Taylor series

We are looking for functions $y(t)$ which solves the simple harmonic oscillator differential equation:

$$y''(t) + y(t) = 0.$$

You can probably guess what the solution to this equation is already without dong any work. However, we can also solve, or at least approximate solutions to, differential equations using Taylor series.

Assume $y(t) = \sum_{n=0}^{\infty} a_n t^n$ is a solution.

Assume $a_0 = 1$ and $a_1 = 0$, what must the rest of the coefficients be? Do you recognize the Taylor series?

What if $a_0 = 0$ and $a_1 = 1$? What function is this the Taylor series to?

Differential equations through Taylor series

We are looking for functions $y(t)$ which solves the simple harmonic oscillator differential equation:

$$y''(t) + y(t) = 0.$$

You can probably guess what the solution to this equation is already without dong any work. However, we can also solve, or at least approximate solutions to, differential equations using Taylor series.

Assume $y(t) = \sum_{n=0}^{\infty} a_n t^n$ is a solution.

Assume $a_0 = 1$ and $a_1 = 0$, what must the rest of the coefficients be? Do you recognize the Taylor series?

What if $a_0 = 0$ and $a_1 = 1$? What function is this the Taylor series to?

In general, if $a_0 = a$ and $a_1 = b$, do you have any guess as to what the function will be?