Today's topics and news

- Topics: Ratio test, power Series

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- Homework: Watch video 14.5 - 14.10

Converge or diverge?

Do the following series diverge or converge?

1. $\sum_{n=2}^{\infty} \frac{1}{n(ln(n))^p}$ where $p > 0$

2. $\sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$

3. $\sum_{n=0}^{\infty} \frac{cos(\pi n)}{n!}$

4. $\sum_{n=1}^{\infty} (\frac{n}{n+1})^n$

5. $\sum_{n=100}^{\infty} \frac{ln(n)}{n^2}$

6. $\sum_{n=1}^{\infty} \frac{sin(n)}{n^2}$

The ratio test

Given a series $\sum_{n=0}^{\infty} a_n$ such that ______________________.

Assume the limit __________________________ exists or is ∞.

1. ____
2. ____
3. ____

The intuition of the ratio test comes from the geometric series. In fact, the proof of the test comes from limit comparison to the the geometric series.

The ratio test

Show that if $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = 1$, then you cannot conclude anything about the series (i.e. the test is inconclusive).

Power Series

A power series is a series (dependent on some variable, ex. x) of the form
$$\sum_{n=0}^{\infty} a_n(x-a)^n$$

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Depending on what x you plug in, the power series might be convergent or divergent. On the x-values where the power series converges, you can think of the power series as representing some function $f(x)$.

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Geometric series

Example: Consider the geometric power series $\sum_{n=0}^{\infty} x^n$. It converges iff $|x| < 1$. On this interval, the power series as a function is equal to the the function $\frac{1}{1-x}$.

New power series from geometric series

Write the power series the following functions and state the interval of convergence.

1. $\frac{1}{1+x}$.
2. $\frac{1}{1-4x^2}$.
3. $\frac{1}{2-x}$.
4. $\ln(1+x)$.
5. $\frac{1}{(1-x)^2}$.
6. $\arctan(x)$.

Interval of convergence

Find the interval of convergence of the following series.

1. $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} x^n$
2. $\sum_{n=1}^{\infty} \frac{2^n}{(2n)!} x^n$
3. $\sum_{n=1}^{\infty} \frac{n^2}{2^n} (x-1)^n$
4. $\sum_{n=1}^{\infty} \frac{1}{n2^n} x^{2n}$.

Examples

Given a series which has a radius of convergence of 1, converges on $(-1,1)$ and satisfies the following extra conditions. If this is impossible, explain why.

1. AC at -1 and AC at 1.
2. AC at -1 and CC at 1.
3. AC at -1 and D at 1.
4. CC at -1 and AC at 1.
5. CC at -1 and CC at 1.
6. CC at -1 and D at 1.
7. D at -1 and AC at 1.
8. D at -1 and CC at 1.
9. D at -1 and D at 1.

Examples

1. Give a series which has a radius of convergence of ∞.
2. Give a series which has a radius of convergence of 0.