Today's topics and news

- Topics: Series tests

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- Homework: Watch video 13.18 - 13.19, 14.1 - 14.4

True or false

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}$ be its partial sum sequence.

10. If $\lim_{n\to\infty} S_{2n}$ exists, then $\sum_{n=0}^{\infty} a_n$ converges.

11. If $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=0}^{\infty} a_n$ converges.

12. If $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{k\to\infty} (\sum_{n=k}^{\infty} a_n) = 0$.

13. If $\sum_{n=0}^{\infty} a_{2n}$ converges and $\sum_{n=0}^{\infty} a_{2n+1}$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

True or false

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10. If $\lim_{n\to\infty} S_{2n}$ exists, then $\sum_{n=0}^{\infty} a_n$ converges. False.

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12. If $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{k\to\infty} (\sum_{n=k}^{\infty} a_n) = 0$. True.

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Relationship between series and type 1 improper integrals

As has been hinted at several times, series can be thought of as a discrete analag of type 1 integrals. As such, lots of the same tests that work for one will work for the other.

For both the convergence of the series and the type 1 integral, it's not enough that the terms a_n or the integrand $f(x)$ to converge to 0 as n or $x \to \infty$. It matters how quickly they are going to 0.

BCT

Basic Comparison Test for Type 1 improper integrals

Given f, g continuous on $[0, \infty)$,

suppose $\forall x \in [0, \infty)$ $0 \leq f(x) \leq g(x)$,

then $\int_0^\infty g(x)dx$ converges $\implies$ $\int_0^\infty f(x)dx$ converges.

Basic Comparison Test for series

Given $\{a_n\}_{n=0}^\infty$, $\{b_n\}_{n=0}^\infty$,

suppose $\forall n \in \mathbb{N}$, $0 \leq a_n \leq b_n$,

then $\sum_{n=0}^\infty b_n$ converges $\implies$ $\sum_{n=0}^\infty a_n$ converges.

Similarly, we have LCT for series:

Basic Comparison Test for series

Given the postive series $\sum_{n=0}^{\infty} a_n$, $\sum_{n=0}^{\infty} b_n$,

suppose $\lim_{n\to\infty} \frac{a_n}{b_n}$ is a positive real number (i.e. $0 < L < \infty$),

then $\sum_{n=0}^{\infty} b_n$ converges $\iff$ $\sum_{n=0}^{\infty} a_n$ converges.

Just like for improper integrals we have extensions to the LCT.

Converge or diverge?

Consider the positive series $\sum_{n=0}^{\infty} a_n$ and suppose $\lim_{n\to\infty} (n!)a_n = 0$. Does $\sum_{n=0}^{\infty} a_n$ necessarily converge?

Suppose instead $\lim_{n\to\infty} na_n = 0$. Does $\sum_{n=0}^{\infty} a_n$ necessarily converge?

Integral test

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Given a continuous function f on $[0, \infty)$.

Suppose f is ___________ and ___________ on $[0, \infty)$,

Suppose $a_n =$ ___________,

then ____.

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If both converge, is it true that $\sum_{n=0}^{\infty} a_n = \int_0^{\infty} f(x)dx$?

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Why is the integral test useful?

If both converge, is it true that $\sum_{n=0}^{\infty} a_n = \int_0^{\infty} f(x)dx$?

If you think back to the proof of this, how does $\sum_{n=0}^{\infty} a_n$ and $\int_0^{\infty} f(x)dx$ compare?

What about between $\sum_{n=1}^{\infty} a_n$ and $\int_0^{\infty} f(x)dx$?

Integral test

Is the following still true?

Claim

Given f continuous, positive and decreasing on $[0, \infty)$, Suppose $a_n = f(n)$, then $\sum_{n=100}^{\infty} a_n$ converges iff $\int_0^{\infty} f(x)dx$ converges.

Is the following still true?

Claim

Given f continuous, positive and decreasing on $[0, \infty)$, Suppose $a_n = f(n)$, then $\sum_{n=100}^{\infty} a_n$ converges iff $\int_0^{\infty} f(x)dx$ converges.

According to the logic above, it seems that we can apply the integral test starting at different points for series and improper integral. So why is it that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges but $\int_0^{\infty} \frac{1}{x^2}dx$ diverges?

Alternating Series

True or false: Is $\sum_{n=1}^{\infty} \sin(n)$ an alternating series?

True or false: Is $\sum_{n=1}^{\infty} \sin(n)$ an alternating series? False.

Give a definition of an alternating series.

AST

Given a series of the form ________________,

If 1. ________________,

2. ________________,

3. ________________,

Then __________________________.

Alternating Series

True or false: Is $\sum_{n=1}^{\infty} \sin(n)$ an alternating series? False.

Give a definition of an alternating series.

AST

Given a series of the form __________________,

If 1. __________________,

2. __________________,

3. __________________,

Then ______________________________.

Alternating Series

The following is easily seen to be an equivalent formulation

AST

Given an alternating series $\sum_{n=0}^{\infty} a_n$,

If $|a_n|$ is decreasing and $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

The following is easily seen to be an equivalent formulation

AST

Given an alternating series $\sum_{n=0}^{\infty} a_n$,

If $|a_n|$ is decreasing and $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=0}^{\infty} a_n$ converges.

Estimating alternating sums

Estimate $S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$ with an error smaller than 0.0001.

Exercise

We saw in PS7 that if $\sum_{n=1}^{\infty} a_n$ is a convergent, non-negative series, then $\sum_{n=1}^{\infty} (a_n)^2$ converges.

Suppose $\sum_{n=1}^{\infty} a_n$ is a convergent series.

Does $\sum_{n=1}^{\infty} (a_n)^2$ necessarily converge or diverge? Does $\sum_{n=1}^{\infty} |a_n|$ necessarily converge or diverge?

Absolute Convergence

We say the series $\sum_{n=0}^{\infty} a_n$ converges absolutely iff ____________________.

We say the series $\sum_{n=0}^{\infty} a_n$ converges conditionally iff ____________________.

True or false: If $\sum_{n=0}^{\infty} a_n$ converges, then it converges absolutely.

True or false: If $\sum_{n=0}^{\infty} a_n$ is an eventually negative series and converges, then it converges absolutely.

True or false: If $\sum_{n=0}^{\infty} a_n$ converges absolutely, then $\sum_{n=0}^{\infty} a_n$ converges.

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Absolute Convergence

We say the series $\sum_{n=0}^{\infty} a_n$ converges absolutely iff $\sum_{n=0}^{\infty} |a_n|$ converges.

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True or false: If $\sum_{n=0}^{\infty} a_n$ converges, then it converges absolutely. False.

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True or false: If $\sum_{n=0}^{\infty} a_n$ converges absolutely, then $\sum_{n=0}^{\infty} a_n$ converges. True.

Positive and negative terms

Let $\sum_{n=0}^{\infty} a_n$ be a series and let b_n be the positive terms (i.e. for each $n \in \mathbb{N}$, $b_n = 0$ if $a_n \leq 0$ and $b_n = a_n$ if $a_n > 0$). Similarly define c_n to be the negative terms.

For each scenario describe the convergence of $\sum_{n=0}^{\infty} a_n$.

1. $\sum_{n=0}^{\infty} b_n$ conv and $\sum_{n=0}^{\infty} c_n$ conv.
2. $\sum_{n=0}^{\infty} b_n$ div and $\sum_{n=0}^{\infty} c_n$ conv.
3. $\sum_{n=0}^{\infty} b_n$ conv and $\sum_{n=0}^{\infty} c_n$ div.

3\. $\sum_{n=0}^{\infty} b_n$ div and $\sum_{n=0}^{\infty} c_n$ div.

Converge or diverge?

Do the following series diverge or converge?

1. $\sum_{n=2}^{\infty} \frac{1}{n(ln(n))^p}$ where $p > 0$
2. $\sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$
3. $\sum_{n=0}^{\infty} \frac{cos(\pi n)}{n!}$
4. $\sum_{n=1}^{\infty} (\frac{n}{n+1})^n$
5. $\sum_{n=100}^{\infty} \frac{ln(n)}{n^2}$
6. $\sum_{n=1}^{\infty} \frac{sin(n)}{n^2}$