Today's topics and news

- Topics: Methods of integration and volume
- Homework: Watch video 11.1 - 11.8
- Term test 3 takes place Monday 6:00 - 8:00 PM. Check BB for more details. Please contact us if you need to arrange an early sitting (at 3:30 - 5:30).

Area of an ellipse

An ellipse with semi-major length a and semi-minor length b can be expressed with the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

1. Using symmetry, write down a formula for the area of an ellipse using integrals.

2. Compute the integral to get a formula for the area of an ellipse.

Integration by trig substitution

Integrate the following:

1. $\int x\sqrt{x^2-9}dx$
2. $\int \frac{1}{x^2\sqrt{x^2+4}}dx$

Integration by partial fraction

Integrate the following:

1. $\int \frac{1}{x^2-3x}dx$.
2. $\int \frac{x^3}{x^2+x-6}dx$.
3. $\int \frac{3x^2+3x+2}{x^3+2x^2+2x}dx$.

Volume by slicing

Consider the region bound by $f(x) = x^2 - 3x + 2$ and the x-axis rotated around the x-axis.

1. What is the cross-section of this solid with the plane orthogonal to the x-axis over $x = 1.5$? What is its area?

2. Let $S(x)$ be the area of the crosssection of this solid with the plane orthogonal the x-axis over x. What is $S(x)$?

3. Make a guess as to how we can calculate the volume of this solid. (Hint: To calculate area, we integrate heights. So how can we calculate volume, a higher dimensional analogue of area?)

$V = \int_a^b S(x)dx$ where the solid is between $x = a$ and $x = b$ and $S(x)$ is the crossectional area over x, so in this case it would be $V = \int_1^2 \pi(x^2 - 3x + 2)^2 dx$.

Volume by slicing

Consider the region bound by $f(x) = x$ and $f(x) = x^2$ rotated around the x-axis. Think about the shape of the crosssetions over each x and calculate the volume of the solid.

Note: Sometimes volume by slicing is called volume by disk and washer. This happens when the crosssections happen to be disks (disks) or annuli (washers).

Volume by slicing

Consider the region bound by $f(x) = x$ and $f(x) = x^2$ rotated around the y-axis.

Is the shape of the crosssections over each x still as clear?

There are two ways to handle the problem in this case.

Method 1: Think about the crosssections over each y instead and try to use volume by slicing.

Volume by cylindrical shells

Consider the region bound by $f(x) = x$ and $f(x) = x^2$ rotated around the y-axis.

1. Think about the line above $x = 0.5$ in the x-y plane bound between the two curves. When you rotate this line around the y-axis, what shape does it become? What's the surface area of that shape?

2. For each $x \in [0, 1]$, find S(x), the surface area of the shape from rotating the line above x in the x-y plane bound between the two curves.

3. Make a guess as to how we can calculate the volume of this solid.

With cylindrical shells, $V = \int_a^b 2\pi r(x)h(x)dx$, so in this case it would be $V = \int_0^1 2\pi x(x - x^2)dx$.

Volume by cylindrical shells

Consider the region bound by $f(x) = x$ and $f(x) = x^2$ rotated around the line $x = -1$.

1. Think about the line above $x = 0.5$ in the x-y plane bound between the two curves. When you rotate this line around the $x = -1$, what shape does it become? What's the surface area of that shape?

2. For each $x \in [0, 1]$, find S(x), the surface area of the shape from rotating the line above x in the x-y plane bound between the two curves.

3. Calculate the volume of this solid.

Integrating along axis

Fill in which method would apply

	dx	dy
Region in x-y plane rotated about x-axis	?	?
Region in x-y plane rotated about y-axis	?	?

Note: Depending on whether your region is described by functions of x or functions of y, one of the two choices of dx or dy might be better.

Volume

Compute the volume of the sphere of radius 1 using:

1. Volume by slicing (disk and washer)
2. Volume by cylindrical shells