Today's topics and news

- Topics: Methods of integration
- Homework: Watch video 9.15 - 9.17

You are going to learn 5 methods of integration (finding antiderivatives) in MAT137:

1. Substitution
2. Integration by parts
3. Integration of trig functions
4. Trig substitution
5. Integration by partial fractions

3 is not really a single method, It's a collection of tricks which helps you find antiderivatives to combinations of trig functions. 4 is also not really a method in its own right, it's really method 1 where you choose to substitute with a trig function. We will go over 1 - 4 today and 5 next week. If you are stuck on an integral, think back to these 5 methods and try out ones you think might be applicable.

Integration by substitution

For substitution, most of the times you are trying to spot one part of the integrand as the derivative of another part. Once you do this it becomes clear what to substitute. There is one example below where this is not the case. Compute:

1. $\int_2^3 \frac{1}{x\sqrt{\ln(x)}}dx.$
2. $\int_0^7 x^2(x+1)^{\frac{1}{3}}dx.$
3. $\int \cot(x)\ln(\sin(x))dx.$
4. $\int_0^1 \frac{x}{1+x^4}dx.$

Odd functions

Given an odd, integrable function $f(x)$ defined on $\mathbb{R}$, show $\forall a \in \mathbb{R}$,

$$\int_{-a}^{a} f(x)dx = 0.$$

Hint: Look at the integral as the sum of integral on two intervals and use substitution to show they cancel. (The fact that they should cancel should tell you what two intervals to choose.)

Integration by parts

Use integration by parts to solve these integrals. Hint: for one of these it's a good idea to do a substitution first.

1. $\int (x^2 + 4x + 1)e^x dx$
2. $\int_1^2 ln(x)dx$
3. $\int x^2 arcsin(x)dx$
4. $\int sin(\sqrt{x})dx$

A reduction formula for $sin^n(x)$

For $n > 2$, prove:

$$\int \sin(x)^n dx = -\frac{1}{n}\sin(x)^{n-1}\cos(x) + \frac{n-1}{n}\int \sin(x)^{n-2}dx.$$

This will help you find $\int \sin(x)^n dx$ inductively as soon as you know what $\int \sin(x)dx$ and $\int \sin(x)^2 dx$ are.

Integration of trig functions

Notice integrals of the form $\int \sin^k(x)\cos(x)dx$, $\int \cos^k(x)\sin(x)dx$ are particularly easy to solve. Why?

Write down the forms for the pairs $(\sec(x), \tan(x))$ and $(\csc(x), \cot(x))$.

If we can use trig identities to reduce our integrals to something of these forms, then we are good. Some formulas you can use are:

1. $\sin^2(x) + \cos^2(x) = 1$
2. $\sin^2(x) = \frac{1-cos(2x)}{2}$
3. $\cos^2(x) = \frac{1+cos(2x)}{2}$

Integration of trig functions

1. $\int \sin^5(x)\cos^2(x)dx$.
2. $\int \sin^2(x)\cos^4(x)dx$.
3. $\int e^{\cos(x)}\cos(x)\sin(x)^5 dx$

Integration of trig functions

Based on the work you've done in the last slide. Let's try to summarize integration strategies in different cases.

For $\int \sin^n(x)\cos^m(x)dx$, $n, m \in \mathbb{N}$

1. If ???, use identity 1 (2 slides prior) and sub $u = \cos(x)$.
2. If ???, use identity 1 and sub $u = \sin(x)$.
3. If ???, use identity 2 and 3 to reduce the powers. Repeat.

For $\int \sec^n(x)\tan^m(x)dx$

1. If ???, use $\sec^2(x) = \tan^2(x) + 1$ and sub $u = \sec(x)$.
2. If ???, use $\sec^2(x) = \tan^2(x) + 1$ and sub $u = \tan(x)$.

Notice our second list does not cover everything.

Integration of trig functions

Integrate:

1. $\int \sec(x)dx$
2. $\int \sec^2(x)dx$
3. $\int \sec^3(x)dx$
4. $\int \sec^4(x)dx$

Area of an ellipse

An ellipse with semi-major length a and semi-minor length b can be expressed with the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

1. Using symmetry, write down a formula for the area of an ellipse using integrals.

2. Compute the integral to get a formula for the area of an ellipse.

Integration by trig substitution

Integrate the following:

1. $\int x\sqrt{x^2 - 9}dx$
2. $\int \frac{x}{\sqrt{x^2+2x+2}}dx$