Today's topics and news

- Topics: Integration by trignometric substitution and by partial fraction
- Homework: None
- Test 3 will cover everything up until this point **and areas (but not volume) which I will cover on Monday**.

Some properties of the integral

1. $\int_a^b f(t)dt + \int_b^c f(t)dt = \int_a^c f(t)dt.$
2. $\int_a^a f(t)dt = 0$
3. $\int_a^b f(t)dt = -\int_b^a f(t)dt$
4. $\int_a^b g(t) + cf(t)dt = \int_a^b g(t)dt + c\int_a^b f(t)dt$
5. If $f(t) \leq g(t)\ \forall t \in [a,b]$ then $\int_a^b f(t)dt \leq \int_a^b g(t)dt$
6. $|\int_a^b f(x)dx| \leq \int_a^b |f(x)|dx$

Trig substitution

1. $\int \sqrt{x^2 - 7} dx$
2. $\int \sqrt{4x^2 + 9} dx$
3. $\int \frac{1}{x^2+2x+5} dx$

The point of integration by partial fraction

1. We can integrate everything of the form: $\frac{A}{Bx+C}$ (in fact of form $\frac{A}{(Bx+C)^k}$), where $A, B, C \in \mathbb{R}$. Why? Convince yourself you can do this by integrating $\frac{2}{3x+4}$ and $\frac{2}{(3x+4)^2}$.

2. We can integrate everything of the form: $\frac{Ax+B}{Cx^2+Dx+E}$ where $Cx^2 + Dx + E$ is an **irreducible quadratic**. This is a bit trickier. Convince yourself you can do this by integrating $\frac{2x+3}{x^2+2x+5}$.

The point of integration by partial fractions is to algebraically change a function of the form $\frac{P(x)}{Q(x)}$ into sums of fractions of the types 1 and 2 (and possibly other types but we won't worry about those for now). As long as your denominator is not too complicated, you can always do this if $\deg(P(x)) < \deg(Q(x))$.

Integration by partial fraction

$$\int \frac{x^5 + 3x^4 + 10x^3 + 11x^2 + 7x + 5}{x^4 + 2x^3 + 5x^2} dx$$