# Announcements

- **Topic:** Indeterminate form and L'Hôpital's Rule
- **Homework:** Watch videos 6.13 and 6.14.

# Backwards L'Hôpital

Construct a polynomial $P$ such that

$$\lim_{x \to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}.$$

# Warm-up: What's the difference?

Which of the following is in indeterminate form?

1. $\lim_{x \to \infty} [x - x]$.
2. $\lim_{x \to \infty} x - \lim_{x \to \infty} x$.

What's the difference?

# Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. $1^{\infty}$
12. $1^{-\infty}$
13. $0^{0}$
14. $0^{\infty}$
15. $0^{-\infty}$
16. $\infty^{0}$
17. $\infty^{\infty}$
18. $\infty^{-\infty}$

# Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions $f$ and $g$ such that

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This is how you show that $\frac{0}{0}$ is an indeterminate form.

2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

3. Prove that $1^{\infty}$, $0^0$, and $\infty^0$ are indeterminate forms. (You will only get $c \geq 0$ this time)

# Computations

Calculate:

1. $\lim\limits_{x\to 2} \dfrac{x^2+2x-6}{x^2+3x-10}$
2. $\lim\limits_{x\to \infty} x^3 e^{-x}$
3. $\lim\limits_{x\to 0} x \sin \dfrac{2}{x}$
4. $\lim\limits_{x\to \infty} x \sin \dfrac{2}{x}$
5. $\lim\limits_{x\to \infty} x \cos \dfrac{2}{x}$
6. $\lim\limits_{x\to 0} \dfrac{e^{2x^2}-\cos x}{x \sin x}$
7. $\lim\limits_{x\to \infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}}$
8. $\lim\limits_{x\to 1} \left[(\ln x)\tan \dfrac{\pi x}{2}\right]$

# Infinity minus infinity

Calculate:

1. $\lim_{x \to 0} \left[ \frac{\csc x}{x} - \frac{\cot x}{x} \right]$
2. $\lim_{x \to \infty} [\ln(x+2) - \ln(3x+4)]$
3. $\lim_{x \to 1} \left[ \frac{2}{x^2 - 1} - \frac{1}{x-1} \right]$
4. $\lim_{x \to -\infty} \left[ \sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right]$

# Exponential indeterminate forms

Calculate:

1. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x-2}\right)^{3x}$
2. $\lim\limits_{x\to 0}\left[1+2\sin(3x)\right]^{4\cot(5x)}$
3. $\lim\limits_{x\to 0^+} x^x$
4. $\lim\limits_{x\to \frac{\pi}{2}^-}(\tan x)^{\cos x}$
5. $\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}\right)^{1/x^2}$