Today's topics and news

- Topic: Integration of rational functions
- **Homework:** Watch videos 10.1 and 10.2 for Tuesday and 11.1 and 11.2 for Wednesday.
- **Test 3** takes place Thursday, Feburary 6th from 4:10 - 6:00 PM. It will cover PL5 - 9 (i.e. up to and including today's lecture).

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If n is even, then use a trig identity to replace all but two of the secs and then try the substitution $u = \tan x$.
- If m is odd, then use a trig identity to replace all but one of the tans and and then try the substitution $u = \sec x$.

Example: you can now integrate $\int \tan(x)dx$ by writing $\tan(x) = \frac{\sin(x)}{\cos(x)}$ or more systematically, by following the idea of the this slide, $\tan(x) = \frac{1}{\sec^2(x)} \sec^2(x) \tan(x) = \frac{1}{1+\tan^2(x)} \sec^2(x) \tan(x)$.

Integral of $\sec(x)$

The previous slide does not cover all cases. For example, the method outlined does not cover $\int \sec(x)$. This particular integral was discussed in video 9.12 and there are two ways to find the integral.

Warm-up: Find $\int \sec(x)dx$. (Hint: multiply and divide $\sec(x)$ by $\sec(x) + \tan(x)$.)

Homework: Find $\int \sec^3(x)dx$ (Hint: Use $\int \sec(x)dx$ and an integration method.)

Rational integrals

1. Calculate $\int \frac{1}{x+a}\,dx$
2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x+3}$
3. Calculate $\int \frac{-x+6}{x^2+3x}\,dx$
4. Calculate $\int \frac{1}{x^2+3x}\,dx$
5. Calculate $\int \frac{1}{x^3-x}\,dx$

Repeated factors

1. Calculate $\int \frac{1}{(x+1)^n}\,dx$ for $n > 1$
2. Calculate $\int \frac{(x+1)-1}{(x+1)^2}\,dx$
3. Calculate $\int \frac{2x+6}{(x+1)^2}\,dx$
4. Calculate $\int \frac{x^2-5}{(x+1)^2}\,dx$
5. How would you calculate $\int \frac{\text{polynomial}}{(x+1)^3}\,dx$?

Irreducible quadratics

1. Calculate $\int \frac{1}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$.
   Hint: You should be able to do these very quickly.
2. Calculate $\int \frac{2x+3}{x^2+1}\,dx$
3. Calculate $\int \frac{x^3}{x^2+1}\,dx$
4. Calculate $\int \frac{x}{x^2+x+1}\,dx$

   Hint: Transform it into one like the previous ones

Messier rational functions

1. How can we compute an integral of the form
$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)}dx \ ?$$

2. How can we compute an integral of the form
$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)x^4(x^2+1)(x^2+4x+7)}dx \ ?$$