Today's topics and news

- Topic: Integration by parts and integration of trig functions
- **Homework:** Watch videos 9.15 (9.16 and 9.17 are supplementary).

Warm-up

Try using integration by parts to integrate the following:

1. $\int xe^{-2x}dx$
2. $\int (x+3)^2 \frac{1}{\sqrt{x+1}}dx$

Computation practice: Integration by parts

Compute

1. $\displaystyle\int xe^{-2x}dx$
2. $\displaystyle\int \ln x \, dx$
3. $\displaystyle\int x \arctan x \, dx$
4. $\displaystyle\int x^2 \arcsin x \, dx$

Exp-trig antiderivative

We want to compute

$$I = \int e^{ax} \sin(bx)\, dx$$

Hint: You will need to use integration by parts twice. Once you get it to work, think about what happens if you made different chocies in your integration by parts.

Persistence

Compute

- $\int_1^e (\ln x)^4 \, dx$
- $\int_1^e (\ln x)^{10} \, dx$

Persistence

There is a more efficient approach. Call

$$I_n = \int_1^e (\ln x)^n \, dx$$

Use integration by parts on I_n. You will get a relationship between I_n and I_{n-1}. Now solve the previous questions.

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (**Once you get them to a form from where it is easy to finish, you may stop.**)

1. $\int \sin^{10} x \cos x \, dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int \cos^2 x \, dx$
4. $\int e^{\cos x} \cos x \sin^5 x \, dx$

Useful trig identities

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

Practice: Integrals with trigonometric functions

1. $\int \sin^{10} x \cos x \ dx$
2. $\int \sin^{10} x \cos^3 x \, dx$
3. $\int \cos^2 x \, dx$
4. $\int e^{\cos x} \cos x \sin^5 x \, dx$

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- What are the two basic forms that are easy to integrate directly with a subsitution?
- If $\boxed{???}$, then try a trig identity and then the substitution $u = \tan x$.
- If $\boxed{???}$, then try a trig identity and then the substitution $u = \sec x$.

Hint: You will need

- $\dfrac{d}{dx}[\tan x] = \dots$
- $\dfrac{d}{dx}[\sec x] = \dots$

- The trig identity involving sec and tan

Integral of products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- What are the two basic forms that are easy to integrate directly with a subsitution?
- If $\boxed{???}$, then use a trig identity and then try the substitution $u = \tan x$.
- If $\boxed{???}$, then use a trig identity and the try the substitution $u = \sec x$.

Integral of $\sec(x)$

Notice the scenarios from the previous slide does not cover some cases. For example, the following:

1. $\int \tan(x)dx$
2. $\int \sec(x)dx$ (Hint: multiply and divide by $\sec(x) + \tan(x)$)
3. $\int \sec^3(x)dx$ (Hint: use 2)

A pair of mysterious functions

Suppose functions $\alpha(x), \beta(x)$ satisfy the following:

1. $\alpha'(x) = 2\beta(x)$
2. $\beta'(x) = \frac{1}{2}\alpha(x)$
3. $\alpha(x)^2 - \beta(x)^2 = 1.$

Do not try to find formulas for $\alpha(x)$ or $\beta(x)$. Integrate the following (your answers will have terms involving α and β and that's fine):

1. $\int \sin(x)\alpha(x)dx$
2. $\int \frac{\beta(x)^3}{\alpha(x)^4}$.