Today's topics and news

- Topic: Area, Integration by substitution
- **Homework:** Watch videos 9.5 - 9.12 (9.7 - 9.9, 9.11 and 9.12 supplementary) for Tuesday and 9.15 - 9.17 (9.16 and 9.17 supplementary) for Wednesday.

Examples of FTC 2 — Areas

Calculate the area of the bounded region...

1. ... between the x-axis and $y = 4x - x^2$.
2. ... between $y = \cos x$, the x-axis, from $x = 0$ to $x = \pi$.
3. ... between $y = x^2 + 3$ and $y = 3x + 1$.
4. ... between $y = 1$, the y-axis, and $y = \ln(x + 1)$.

Warm up

Calculate

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Definite integral via substitution

This final answer is right, but the write-up is WRONG. Why?

Calculate $I = \int_0^2 \sqrt{x^3+1}\, x^2 dx$

Wrong answer

Substitution: $u = x^3 + 1,\ du = 3x^2 dx.$

$$\begin{aligned} I &= \frac{1}{3}\int_0^2 \sqrt{x^3+1}\,(3x^2dx) & &= \frac{1}{3}\int_0^2 u^{1/2}\, du \\ &= \frac{1}{3}\,\frac{2}{3}\ \left. u^{3/2}\right|_0^2 & &= \left.\frac{1}{9}\left(x^3+1\right)^{2/3}\right|_0^2 \\ &= \frac{2}{9}\left(2^3+1\right)^{3/2} - \frac{2}{9}\left(0+1\right)^{3/2} & &= \frac{52}{9} \end{aligned}$$

Computation practice: integration by substitution

Guess the substitution you want to use (don't actually substitute):

1. $$\int e^x \cos\left(e^x\right) dx$$
2. $$\int \cot x \, dx$$
3. $$\int x^2 \sqrt{x+1} \, dx$$
4. $$\int \frac{e^{2x}}{\sqrt{e^x+1}} \, dx$$
5. $$\int \frac{(\ln \ln x)^2}{x \ln x} \, dx$$
6. $$\int x e^{-x^2} \, dx$$

A different kind of substitution

Calculate

$$\int_0^1 \sqrt{1 - x^2}\, dx$$

using the substitution $x = \sin\theta$.

This is often called integration by trignometric substitution, which is covered in videos 9.13 and 9.14.

Odd functions

Theorem

Let f be a continuous function. Let $a > 0$. IF f is odd, THEN

$$\int_{-a}^{a} f(x)dx = ???$$

1. Draw a picture to interpret the theorem geometrically.
2. Write down the definition of "odd function".
3. Prove the theorem!
   Hint: Write the integral as sum of two pieces. Use a substitution in one of them to show that they cancel with each other.