Today's topics and news

- Topic: Graph sketching
- **Homework:** Watch videos 7.1 - 7.4 for Tuesday and 7.5 - 7.7 for Wednesday of the week you are back. More improtantly have a relaxing winter break!

An equation from the asymptotes

Construct a function f that satisfies all the following conditions at the same time.

- f is a rational function (this means it is a quotient of polynomials).
- The line $y = 1$ is an asymptote of the graph of f.
- The line $x = -1$ is an asymptote of the graph of f.

Periodic?

Construct a function H that has all the following properties at once:

1. The domain of H is $\mathbb{R}$
2. H is strictly increasing on $\mathbb{R}$
3. H is differentiable on $\mathbb{R}$
4. H' is periodic with period with period 2
5. H' is not constant

Even and odd functions

Even function

A function is even if $\forall x \in$ doman of f, $-x$ is in the domain of f and $f(x) = f(-x)$.

Odd function

A function is even if $\forall x \in$ doman of f, $-x$ is in the domain of f and $-f(x) = f(-x)$.

These are not dichotomous - most functions are neither even nor odd!

A weird function

The function $G(x) = xe^{1/x}$ is deceiving. To help you out:

$$G'(x) = \frac{x-1}{x}e^{1/x}, \qquad G''(x) = \frac{e^{1/x}}{x^3}$$

1. Carefully study the behaviour as $x \to \pm\infty$.
   You should find an asymptote, but it is not easy.
2. Carefully study the behaviour as $x \to 0^+$ and $x \to 0^-$.
   The two are very different.
3. Use G' to study monotonocity.
4. Use G'' to study concavity.
5. Sketch the graph of G.

A note on the previous slide

1. Even though monotonicity and concavity changes between $x > 0$ and $x < 0$, 0 should not be considered a critical point or an inflection point. Why?
2. One should check if $G(x)$ intersects $y = x + 1$. It doesn't - when $x > 0$, $G(x) > x + 1$ always. You can justify this by checking that $G(x) - (x + 1)$ is decreasing for $x > 0$. Since $G(x) - (x + 1)$ limits to 0 as $x \to \infty$ and ∞ as $x \to 0^+$, we can conclude $G(x) - (x + 1) > 0$ for $x > 0$.