Today's topics and news

- Topic: Indeterminate forms and L'Hôpital's Rule
- **Homework:** Watch videos 6.1 and 6.2 for Wednesday.
- **Reminder:** There will be a formula sheet for trig identities on the test so don't stress about memorizing them. Please review the vocabulary sheet for the test.

An interesting function

Let

$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

1. Calculate $f'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $f'(0)$.
3. Is f continuous at 0?
4. Is f differentiable at 0?
5. Is f' continuous at 0?

Warm-up: What's the difference?

Which of the following is in indeterminate form?

1. $\lim_{x\to\infty} [x - x]$.
2. $\lim_{x\to\infty} x - \lim_{x\to\infty} x$.

What's the difference?

Indeterminate?

Which of the following are indeterminate forms for limits?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

   This is how you show that $\frac{0}{0}$ is an indeterminate form.

2. Show the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

3. Show that 1^∞, 0^0, and ∞^0 are indeterminate forms. (You will not be able to get all $c \in \mathbb{R}$ this time.)

What's wrong with the following computation?

Since $\lim\limits_{x\to\infty} \frac{x+\sin(x)}{x}$ is in indeterminate form,

$\lim\limits_{x\to\infty} \frac{x+\sin(x)}{x} = \lim\limits_{x\to\infty} \frac{1+\cos(x)}{1}$ by LH.

Therefore, $\lim\limits_{x\to\infty} \frac{x+\sin(x)}{x}$ DNE since $1+\cos(x)$ oscillates between 0 and 2 as $x \to \infty$.

What does $\lim\limits_{x\to\infty} \frac{x+\sin(x)}{x}$ actually equal?

Infinity minus infinity

Compute:

1. $\lim_{x\to\infty} [\ln(x+2) - \ln(3x+4)]$
2. $\lim_{x\to-\infty} \left[\sqrt{x^2+3x} - \sqrt{x^2-3x}\right]$
3. $\lim_{x\to 0} \left[\frac{\csc x}{x} - \frac{\cot x}{x}\right]$
4. $\lim_{x\to 1} \left[\frac{2}{x^2-1} - \frac{1}{x-1}\right]$

Exponential indeterminate forms

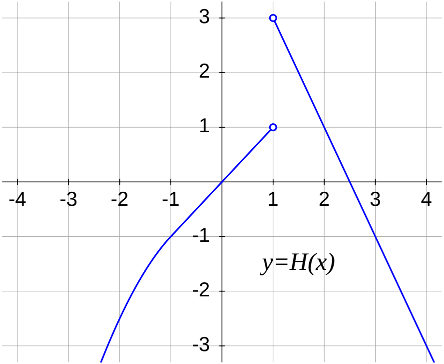
Compute:

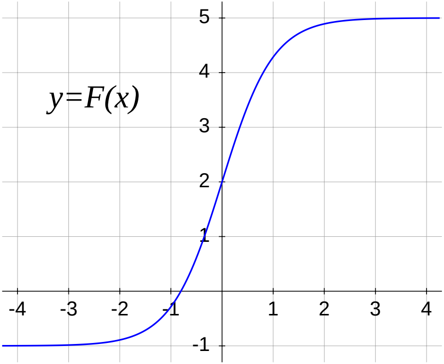
1. $$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$$
2. $$\lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x}$$
3. $$\lim_{x \to 0} [1 + 2\sin(3x)]^{4\cot(5x)}$$
4. $$\lim_{x \to \infty} \left(\frac{x+2}{x-2}\right)^{3x}$$
5. $$\lim_{x \to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$$

Compute:

1. $\lim_{x \to 0} \dfrac{H(x)}{H(2+3x)-1}$

2. $\lim_{x \to 2} \dfrac{F^{-1}(x)}{x-2}$





Backwards L'Hôpital

1. Construct a polynomial P such that
$$\lim_{x\to 1} \frac{P(x)}{e^x - e \cdot x} = \frac{1}{e}$$
2. Find $a \in \mathbb{R}$ and $n \in \mathbb{N}$ such that the limit
$$\lim_{x\to 0} \frac{\sin x - ax^n}{x^3}$$
exists. What is the value of the limit?