Today's topics and news

- Topic: Monotonicity
- **Homework:** Watch videos 6.3 - 6.10 for Tuesday and 6.1, 6.2 for Wednesday.

Increasing functions

Given interval $\mathbb{I}$ and f defined on $\mathbb{I}$.

Give the definition for "f is increasing on $\mathbb{I}$".

Theorem

Let $a < b \in \mathbb{R}$.

Let f differentiable on (a, b).

IF $\forall x \in (a, b)$, $f'(x) > 0$.

THEN f is increasing on (a, b).

Is this proof OK?

Theorem

Let $a < b \in \mathbb{R}$.

Let f differentiable on (a, b).

IF $\forall x \in (a, b)$, $f'(x) > 0$.

THEN f is increasing on (a, b).

Proof: Assume $\forall x \in (a, b), f'(x) > 0$.

Let $x_1, x_2 \in (a, b)$. Assume $x_2 > x_1$.

Since $f'(x_1) > 0$, we have $\lim\limits_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0$.

Therefore, $\frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0$.

Since $x_2 - x_1 > 0$, we have $f(x_2) - f(x_1) > 0$ (i.e. $f(x_2) > f(x_1)$ as required.)

True or false?

Theorem

Let $a < b \in \mathbb{R}$.

Let f differentiable on (a, b).

IF $\forall x \in (a, b)$, $f'(x) < 0$.

THEN f is decreasing on $[a, b]$.

Proving difficult identities

Prove that, for every $x \geq 0$,

$$\arcsin \frac{1-x}{1+x} + 2 \arctan \sqrt{x} = \frac{\pi}{2}$$

Hint: Take derivatives.

Intervals of monotonicity

Let $g(x) = x^3(x^2 - 4)^{1/3}$.

Find out on which intervals this function is increasing or decreasing.
Using that information, sketch its graph.

To save time, here is the first derivative:

$$g'(x) = \frac{x^2(11x^2 - 36)}{3(x^2 - 4)^{2/3}}$$

Inequalities

Prove that, for every $x \in \mathbb{R}$

$$e^x \geq 1 + x$$

Hint: When is the function $f(x) = e^x - 1 - x$ increasing or decreasing?