Today's topics and news

- Topic: Extrema, Rolle's Theorem, MVT
- **Homework:** Watch videos 5.10 - 5.12 for Wednesday.
- **PSB** has been posted.
- **Test 2** will take place November 29th from 4:10 - 6:00. Please look on the course website for more details. It will cover up to and until this Wednesday's lecture.

Why doesn't this argument work?

Consider the function

$$f(x) = \begin{cases} x^2 & x \neq 0 \\ 1 & x = 0 \end{cases}$$

Notice the only critical point of f is $x = 0$. To the left of this critical point, $f'(x) < 0$ and to the right of this critical point, $f'(x) > 0$.

Therefore, f has a local minimum at $x = 0$.

Extrema on a domain of $\mathbb{R}$

Let $h(x) = x^4 - 2x^2$.

Find the global extrema of h on $\mathbb{R}$.

Zeroes of the derivative

If possible, construct a function f that is differentiable on $\mathbb{R}$ and such that

1. f has exactly 2 zeroes and f' has exactly 1 zero.
2. f has exactly 2 zeroes and f' has exactly 2 zeroes.
3. f has exactly 3 zeroes and f' has exactly 1 zero.
4. f has exactly 1 zero and f' has infinitely many zeroes.

How many zeroes?

Let

$$f(x) = x^2 - \cos(x)$$

How many zeroes does f have?

Let

$$g(x) = x^2 + \cos(x)$$

How many zeroes does g have?
Do this question without using any graphing utilities.

A corollary to Rolle's Theorem

Prove:

Theorem 1

Let f be a differentiable function defined on an open interval I.
IF $\forall x \in I, f'(x) \neq 0$
THEN f is one-to-one on I.

Roots of a polynomial

Given $n \in \mathbb{N}$.

A polynomial of degree n is a function $P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_0$ where $\forall i = 0, ..., n,\ a_i \in \mathbb{R}$ and $a_n \neq 0$.

Prove a polynomial of degree n can have at most n roots.

Increasing functions

Given interval $\mathbb{I}$ and f defined on $\mathbb{I}$.

Give the definition for "f is increasing on $\mathbb{I}$".

Theorem

Let $a < b \in \mathbb{R}$.

Let f differentiable on (a, b).

IF $\forall x \in (a, b)$, $f'(x) > 0$.

THEN f is increasing on (a, b).