Today's topics and news

- Topic: one-to-one functions, inverse trig functions

Today's topics and news

- Topic: one-to-one functions, inverse trig functions
- **Homework:** Watch videos 5.1 - 5.4 for Wednesday.

Composition and inverses

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Let f and g be functions. Assume they each have an inverse.

Is $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$?

- If YES, prove it.
- If NO, fix the statement.

If you do not know how to start, experiment with the functions

$$f(x) = x + 1, \qquad g(x) = 2x.$$

Composition of one-to-one functions – 2

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Is the following claim TRUE or FALSE? Prove it or give a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one,
THEN g is one-to-one.

Derivative of the inverse

Let f be one-to-one.

Let $a, b \in \mathbb{R}$ s.t. $f(a) = b$.

Suppose both f and f^{-1} are three times differentiable.

1. Find a formula for $(f^{-1})'(b)$ involving $f'(a)$.
2. Find a formula for $(f^{-1})''(b)$ involving $f'(a)$ and $f''(a)$.
3. Find a formula for $(f^{-1})'''(b)$.

Composition of one-to-one functions – 3

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

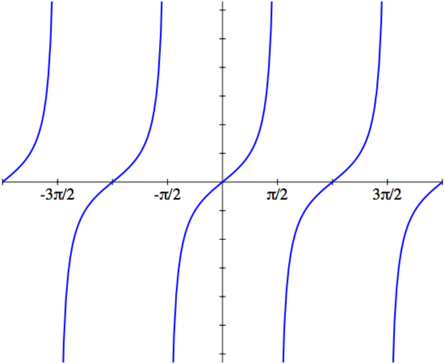
Is the following claim TRUE or FALSE? Prove it or give a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one,
THEN f is one-to-one.

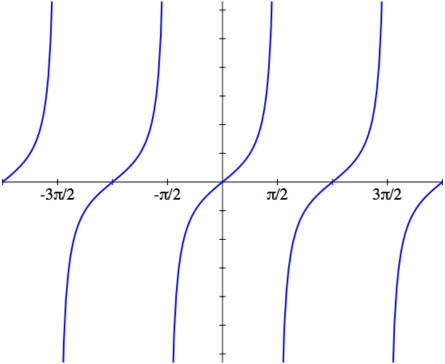
The arctan function

Here's (part of) the graph of the tan function.



The arctan function

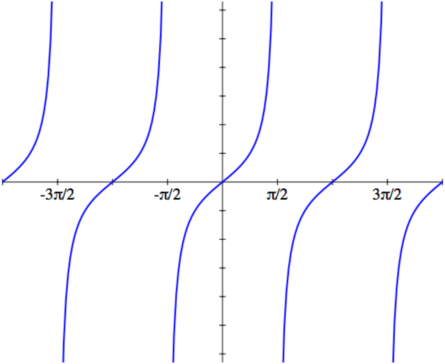
Here's (part of) the graph of the tan function.



Question. Does this function have an inverse?

The arctan function

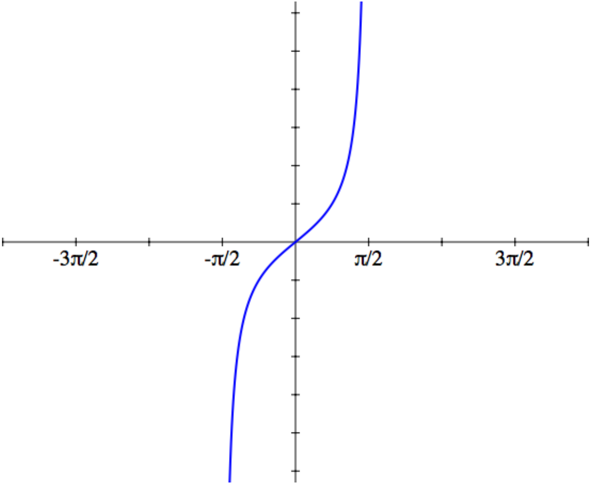
Here's (part of) the graph of the tan function.



Question. Does this function have an inverse?
Problem. Find the largest interval containing 0 such that the restriction of tan to it is injective.

The arctan function

We define arctan to be the inverse of the function with this graph:



The arctan function

In symbols, that means we define the function arctan as the inverse of the function

$$g(x) = \tan x, \text{ restricted to the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

The arctan function

In symbols, that means we define the function arctan as the inverse of the function

$$g(x) = \tan x, \text{ restricted to the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

In other words, if $x, y \in \mathbb{R}$, then

$$\arctan(y) = x \quad \Longleftrightarrow \quad \begin{cases} ??? \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ ??? \end{cases}$$

Problem 1. What should be where the question marks are?

The arctan function

In symbols, that means we define the function arctan as the inverse of the function

$$g(x) = \tan x, \text{ restricted to the interval } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

In other words, if $x, y \in \mathbb{R}$, then

$$\arctan(y) = x \quad \Longleftrightarrow \quad \begin{cases} ??? \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ ??? \end{cases}$$

Problem 1. What should be where the question marks are?

Problem 2. What are the domain and range of arctan?

Problem 3. Sketch the graph of arctan.

The arctan function

To remind you:

$$\arctan(y) = x \quad \Longleftrightarrow \quad \begin{cases} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \tan x = y \end{cases}$$

The arctan function

To remind you:

$$\arctan(y) = x \iff \begin{cases} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \tan x = y \end{cases}$$

Compute the following values:

1. $\arctan\left(\tan\left(1\right)\right)$
2. $\arctan\left(\tan\left(3\right)\right)$
3. $\arctan\left(\tan\left(\frac{\pi}{2}\right)\right)$
4. $\arctan\left(\tan\left(-6\right)\right)$
5. $\tan\left(\arctan\left(0\right)\right)$
6. $\tan\left(\arctan\left(10\right)\right)$

Find $\frac{d}{dx}\arctan(x)$.

Hint: You should simplify your answer so that it doesn't have any trig/inverse trig functions in them.