- Topic: Implicit differentiation, exponential and logarithms, related rates

- **Homework:** Watch videos 4.1 and 4.2 for Wednesday

Implicit differentiation

The equation
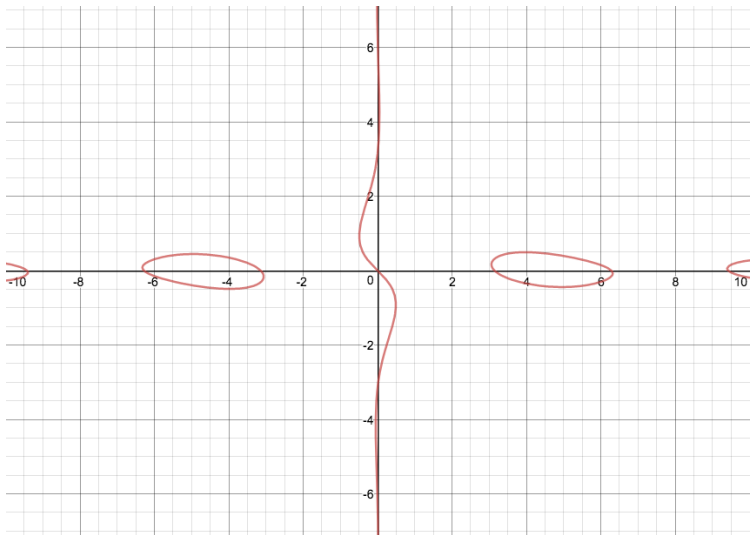$$\sin(x + y) + xy^2 = 0$$
defines a function $y = h(x)$ near $(0, 0)$.

Compute:

1. $h(0)$

2. $h'(0) = \left.\dfrac{dy}{dx}\right|_{x=0, y=0}$

3. $h''(0) = \left.\dfrac{d^2y}{dx^2}\right|_{x=0, y=0}$

4. $h'''(0) = \left.\dfrac{d^3y}{dx^3}\right|_{x=0, y=0}$

Implicit differentiation

1. What is $\frac{dx}{dy}\bigg|_{x=0, y=0}$? Make a guess from your previous work and check it by implicit differentiation.

2. What is $\frac{d^2x}{dy^2}\bigg|_{x=0, y=0}$?

Exercise: Derivative of the other trig functions

Using the differentiation rules and

$$\frac{d}{dx}\sin(x) = \cos(x), \quad \frac{d}{dx}\cos(x) = -\sin(x).$$

Find:

1. $\frac{d}{dx}\tan(x)$
2. $\frac{d}{dx}\cot(x)$
3. $\frac{d}{dx}\sec(x)$
4. $\frac{d}{dx}\csc(x)$

Compute the derivatives of the following:

1. $\ln[e^x + \ln(\ln(x))]$
2. $2^{\tan(x)}$
3. $\log_{x+1}(x^2 + 1)$ Hint: Try using a log identity first.

Multiple choice

The derivative of x^x is:

a. x^{x-1}

b. $ln(x)x^x$

c. $(ln(x) + 1)x^x$

Logrithmic differentiation

Find $\frac{dy}{dx}$:

1. $y = x^{x^x} + 1$

2. $x^y = x^2 + y^x$

Related rates

Typical related rates problems: If there is a relationship between quantities and you know how one quantity is changing (usually with respect to time), then how does the other quantity change?

Example: You are filling up a perfectly spherical balloon. You inflate it at a rate of 1000 cm^3/s. At what rate is the radius of the balloon changing when the radius of the balloon is 20cm?

The formula for the volume of a sphere is $V = \frac{4\pi r^3}{3}$.

A 10-meter long ladder is leaning against a vertical wall and sliding. The top end of the ladder is 8 meters high and sliding down at a rate of 1 meter per second. At what rate is the bottom end sliding away from the wall?

Math Halloween party

The MAT137 TAs wanted to rent a disco ball for their upcoming Halloween party. However, since they are poor, they can only afford a flashlight. At the party, one TA is designated the "human disco ball". This TA stands in the centre of the room pointing the flashlight horizontally and spins at 3 revolutions per second. (YES, they are THAT fast.) The room is square with side length 8 meters. At what speed is the light from the flashlight moving across the wall when it is 2 meters away from a corner?

An interesting example

Let $f(x) = x^2 \sin \dfrac{1}{x}$.

1. Calculate $f'(x)$ for any $x \neq 0$.
2. Using the definition of derivative, calculate $f'(0)$.
3. Is f continuous at 0?
4. Is f differentiable at 0?
5. Is f' continuous at 0?