Today's topics and news

- Topic: Chain rule, trig functions, implicit differentiation
- **Homework:** Watch videos 3.13 - 3.20 for Tuesday and 4.1, 4.2 for Wednesday.

Two different limits

Consider a function f differentiable everywhere. Compute the following limits in terms of $f'(x)$.

1. $\displaystyle\lim_{h\to 0} \frac{f(2x+h)-f(2x)}{h}$
2. $\displaystyle\lim_{h\to 0} \frac{f(2(x+h))-f(2x)}{h}$

Assume f and g have derivatives of all order.

Find formulas for:

1. $(f \circ g)'(x)$
2. $(f \circ g)''(x)$
3. $(f \circ g)'''(x)$

in terms of the values of f, g and their derivatives of any order.

Compute the derivative of $\cos(x)$ from the definition of derivative as a limit.

Hint: Write down the limit and try to imitate what was done for $\sin(x)$ in the videos. If you need a trig identity that you do not know, google it or ask your neighbour.

Implicit Differentiation

Function: For each input there is a unique output

Relation: A relationship between several variables with no well-defined idea of an input and an output, in particular no "uniqueness" of output.

Example: $x^2 + y^2 = 1$ is a relation but not a function.

We can still graph the relation by drawing the curve(s) of all (x, y) satisfying the equation and talk about the "tangent slope" at a given point on the graph.

However, saying something like find $\left.\frac{dy}{dx}\right|_{x=0}$ (usually) doesn't make sense. Why?

Implicit differentiation

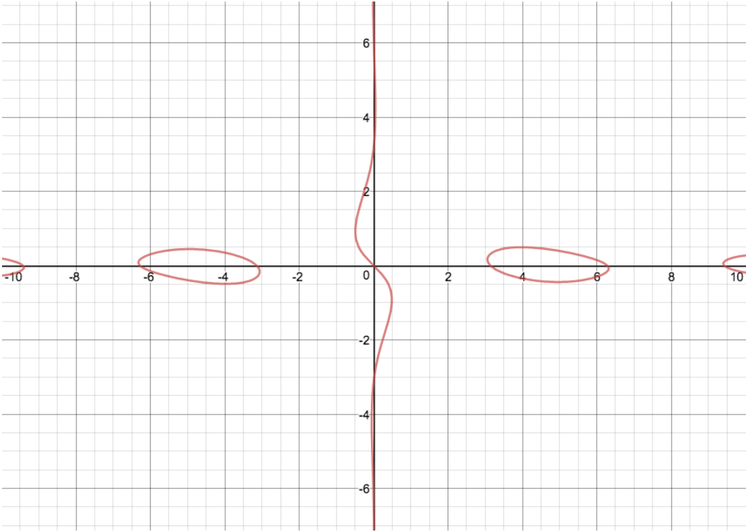
The equation

$$\sin(x+y) + xy^2 = 0$$

defines a function $y = h(x)$ near $(0,0)$.

Compute:

1. $h(0)$
2. $h'(0) = \left.\frac{dy}{dx}\right|_{x=0,y=0}$
3. $h''(0) = \left.\frac{d^2y}{dx^2}\right|_{x=0,y=0}$
4. $h'''(0) = \left.\frac{d^3y}{dx^3}\right|_{x=0,y=0}$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Implicit differentiation

1. What is $\left.\frac{dx}{dy}\right|_{x=0,y=0}$? Make a guess from your previous work and check it by implicit differentiation.

2. What is $\left.\frac{dx^2}{dy^2}\right|_{x=0,y=0}$?

Exercise: Derivative of the other trig functions

Using the differentiation rules and

$$\frac{d}{dx}\sin(x) = \cos(x), \qquad \frac{d}{dx}\cos(x) = -\sin(x).$$

Find:

1. $\frac{d}{dx}\tan(x)$
2. $\frac{d}{dx}\cot(x)$
3. $\frac{d}{dx}\sec(x)$
4. $\frac{d}{dx}\csc(x)$