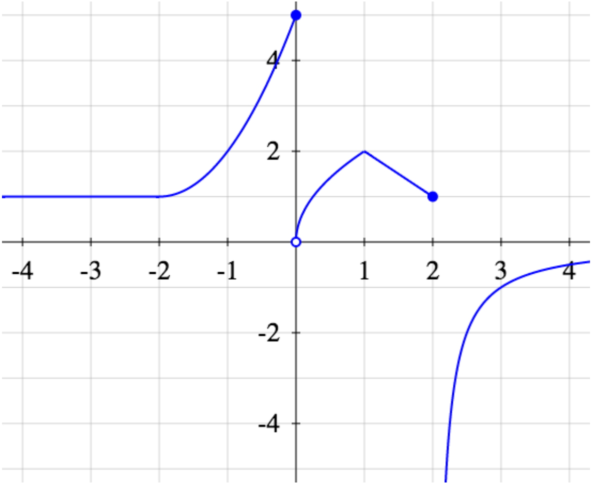
Today's topics and news

- Topic: Chain rule, Proof of differentiation rules
- **Homework:** Watch videos 3.11 and 3.12 for Wednesday.

Intuitive idea of the derivative

Graph the derivative of this function.



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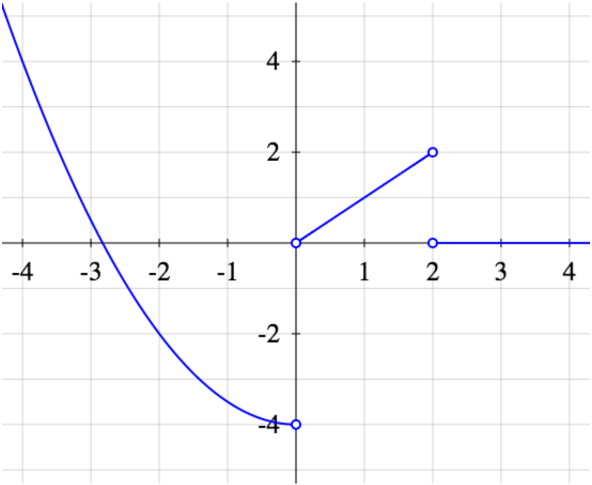
Below is the graph of the derivative of some function f. We know f is continuous and $f(0) = 0$. Graph f.



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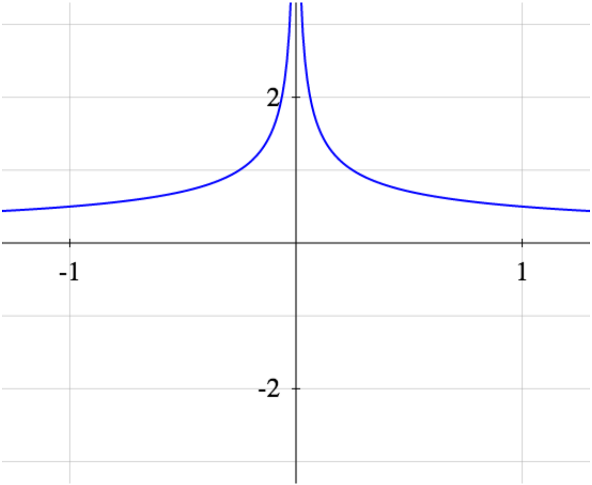

Warm-up: differentiability implies continuity

Let $a \in \mathbb{R}$.

Let f be defined in a neighbourhood of a.

Write the definitions of "f is continuous at a" and "f is differentiable at a" using limits.

1. Prove if f is differentiable at a then f is continuous a.

2. Show it's not necessarily true that if f is continuous a then f is differentiable a.

A lemma

Let $a \in \mathbb{R}$.

Given a function f defined in a neighbourhood of a.

Assume f is continuous at a and $f(a) \neq 0$.

Prove $\exists \delta > 0$ s.t. $\forall x \in (a-\delta, a+\delta)$, $f(x) \neq 0$.

Quotient rule

Let $a \in \mathbb{R}$.

Given functions f and g defined in a neighbourhood of a.

Define $h(x) = \frac{f(x)}{g(x)}$.

Assume f and g are _____________.

Assume ______________________.

Then ______________________ and

____.

Prove this.

Grade your partner's proof (out of 8)

1. [1] Did they check $h(x) = \frac{f(x)}{g(x)}$ is actually defined in a neighbourhood of a. (Is it necessary to check this?)

2. [1] Did they start by using the definition of derivatives for h?

3. [1] Can you understand all the steps clearly without having to guess at their meaning?

4. [2] Did they assume at some point a function is differentiable? If so, did they justify it?

5. [1] Did they assume at some point a function is continuous? If so, did they justify it? (This has to come up in the proof somewhere.)

6. [2] Does the proof work?

Warm-up

Compute the derivative of the following (do not worry too much about the domain):

1. $f(x) = \sqrt{2x^2 + x + 1}$
2. $g(x) = \sqrt{x + \sqrt{x + \sqrt{x + 1}}}$