Today's topics and news

- Topic: More proofs, limit computations
- **Homework:** Watch videos 2.21, 2.21, and 3.1 - 3.3 for next Tuesday and 3.4, 3.5 and 3.8 for Wednesday.
- **Test 1** covers up until and including today's lecture (i.e. video 2.20).
- **PS A** has been posted. It is for practice only. I strongly recommend that you work on it in preparation for the test.

Behaviour of limits under composition

Yesterday we saw this theorem:

Theorem: limit "commutes" with continuous functions

IF $\lim_{x \to a} g(x)$ exists and f is continuous at $\lim_{x \to a} g(x)$.
THEN $\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x))$.

We might want to know what happens if f is not continuous and there are in fact certain scenarios where we can still say something.

Behaviour of limits under composition

Let f be a function defined on an open neighbourhood of 10, except possibly at 10.

Suppose $\lim_{x \to 10} f(x) = 2$.

What can you say about the following:

1. $\lim_{x \to 2} f(5x)$?
2. $\lim_{x \to 2} 5f(x)$?

Notice (1) is not an application of yesterday's theorem because we are not assuming continuity of $f(x)$.

Behaviour of limits under composition

Fill in the blank and then prove the claim.

Claim

Let $a, L \in \mathbb{R}$.

Let f be a function defined on a punctured neighbourhood of a (i.e on some open neighbourhood of a, except possibly at a).

If $\lim_{x \to a} f(x) = L$

Then $\lim_{x \to \frac{a}{5}} 2f(5x) =$ ______________.

Computations

Suppose $\lim_{x \to a} f(x) = L$, then $\lim_{x \to \frac{a}{k}} f(kx) = L$.

Compute:

1. $\lim_{x \to 0} \frac{\sin(3x)}{x}$
2. $\lim_{x \to 0} \frac{1 - \cos(x)}{x}$

Computations using limit laws

Given a function g s.t.

$$\lim_{x\to 0}\frac{g(x)}{x^2}=2.$$

Use it to compute the following limits (or explain that they don't exist).

1. $\lim_{x\to 0}\frac{g(x)}{x}$
2. $\lim_{x\to 0}\frac{g(x)}{x^4}$
3. $\lim_{x\to 0}\frac{g(3x)}{x^2}$

Computations

Compute:

1. $\lim_{x \to 2} \frac{|x^2-4|}{x^2-5x+6}$
2. $\lim_{x \to 4} \frac{x^2-5x+4}{\sqrt{x}-2}$
3. $\lim_{x \to \infty} \frac{x^3+\sqrt{2x^6+1}}{2x^3+\sqrt{x^5+1}}$
4. $\lim_{x \to -\infty} x - \sqrt{x^2+x}$
5. $\lim_{x \to -\infty} x + \sqrt{x^2+x}$