Today's topics and news

- Topic: Squeeze theorem, more limit proofs
- **Homework:** Watch videos 2.14 - 2.18 for next Tuesday. Watch videos 2.19 and 2.20 for next Wednesday.
- **Reminder:** PS2 is due next Thursday.

Limit laws warm-up

Are the following the same?

1. $\lim_{x \to 0^+} \frac{1}{x} - \lim_{x \to 0^+} \frac{1}{x}$
2. $\lim_{x \to 0^+} (\frac{1}{x} - \frac{1}{x})$

Indeterminate form

Let $a \in \mathbb{R}$.

Let f and g be functions defined near a.

Assume $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$.

What can we conclude about $\lim_{x \to a} \frac{f(x)}{g(x)}$?

1. The limit is 1 because the 0s cancel.
2. The limit does not exist because it's $\frac{0}{0}$.
3. We do not have enough information to decide.

True or False?

True or false?

Claim

Let $a \in \mathbb{R}$.

Let f and g be functions defined near a.

- IF $\lim_{x \to a} f(x) = 0$,
- THEN $\lim_{x \to a} [f(x)g(x)] = 0$.

Theorem

Definition

Given a function f defined on some domain $D \subseteq \mathbb{R}$.
We say f is bounded iff $\exists M \in \mathbb{R}$ s.t. $\forall x \in D$, $|f(x)| < M$.

Prove the following claim:

Claim

Let $a \in \mathbb{R}$.
Let f and g be functions defined near a. Let g be bounded.

- IF $\lim\limits_{x \to a} f(x) = 0$,
- THEN $\lim\limits_{x \to a} [f(x)g(x)] = 0$.

Hint: Bound $f(x)g(x)$ by appropriate functions, use the squeeze theorem.

Another squeeze theorem

Another squeeze theorem

Let $a \in \mathbb{R}$. Let f and g be functions defined near a, except possibly at a.

IF

- For x close to a but not a, $f(x) \geq g(x)$.
- $\lim_{x \to a} g(x) = \infty$.

THEN

- $\lim_{x \to a} f(x) = \infty$.

Another squeeze theorem

Another squeeze theorem

Let $a \in \mathbb{R}$. Let f and g be functions defined near a, except possibly at a.

IF

- $\exists p > 0$ s.t. $0 < |x - a| < p \implies f(x) \geq g(x)$.
- $\lim_{x \to a} g(x) = \infty$.

THEN

- $\lim_{x \to a} f(x) = \infty$.

Prove this theorem.

Hint: The proof of this theorem is similar to but easier than the standard squeeze theorem. Write down the relevant $M - \delta$ definitions and try to prove one from the other.