Today's topics and news

- Topic: Definition of a limit
- **Homework:** Watch videos 2.7 - 2.11 for next Tuesday. Watch videos 2.12 and 2.13 for next Wednesday.

Floor

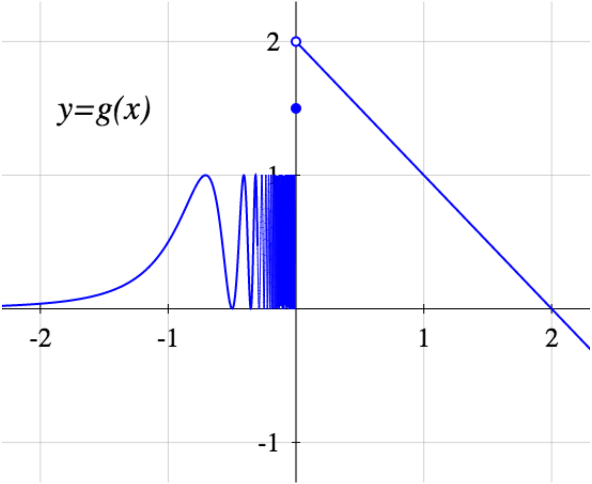
Given a real number x, we defined the *floor of* x, denoted by $\lfloor x \rfloor$, as the largest integer smaller than or equal to x. For example:

$$\lfloor \pi \rfloor = 3, \qquad \lfloor 7 \rfloor = 7, \qquad \lfloor -0.5 \rfloor = -1.$$

Sketch the graph of $y = \lfloor x \rfloor$. Then compute:

1. $\lim_{x \to 0^+} \lfloor x \rfloor$
2. $\lim_{x \to 0^-} \lfloor x \rfloor$
3. $\lim_{x \to 0} \lfloor x \rfloor$
4. $\lim_{x \to 0} \lfloor x^2 \rfloor$

More limits from a graph



Find the value of

1. $\lim_{x\to 0^+} g(x)$
2. $\lim_{x\to 0^+} \lfloor g(x) \rfloor$
3. $\lim_{x\to 0^+} g(\lfloor x \rfloor)$
4. $\lim_{x\to 0^-} g(x)$
5. $\lim_{x\to 0^-} \lfloor g(x) \rfloor$
6. $\lim_{x\to 0^-} \lfloor \frac{g(x)}{2} \rfloor$
7. $\lim_{x\to 0^-} g(\lfloor x \rfloor)$

Formal definition of a limit

Definition of a limit

Given a, $L \in \mathbb{R}$ and

f a function defined in an open interval around a, except possibly at a,

we say that $\lim_{x \to a} f(x) = L$ iff

$\forall \epsilon > 0$, $\exists \delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$.

Translation

Translation of $\forall\epsilon > 0$, $\exists\delta > 0$ s.t. $0 < |x - a| < \delta \implies |f(x) - L| < \epsilon$.

$\forall\epsilon > 0$	"If I give you any distance ϵ..."
$\exists\delta > 0$ s.t.	"... you can find a distance δ such that..."
$0 < |x - a| < \delta \implies$	"... if x is within δ of (but not equal to) a..."
$|f(x) - L| < \epsilon$.	"... then $f(x)$ is within ϵ of L."

Formal definition of a one-sided limit

Given a ,$L \in \mathbb{R}$.

Write down the definition of $\lim_{x \to a^+} f(x) = L$.

Exercise: Write down the definition of $\lim_{x \to a^-} f(x) = L$.

Existence of limits

Given $a \in \mathbb{R}$.

Write down the definition of the following statments:

1. $\lim_{x \to a} f(x)$ exists.
2. $\lim_{x \to a} f(x)$ does not exist.

Existence of limits

Given $a, L \in \mathbb{R}$.

Write down the definition of the following statments:

1. $\lim_{x \to a} f(x) = \infty$.
2. $\lim_{x \to \infty} f(x) = L$.

Hint: For 1, you want to replace the two parts with ϵ in the $\epsilon - \delta$ definition for limits. Instead of saying $f(x)$ gets arbitrarily close to L as x gets close to a, you want to say $f(x)$ gets arbitrarily large. How can you do this?