Today's topics and news

- Topic: Direct proofs and proofs by induction
- **Homework:** Watch videos 2.1 - 2.4 for next Tuesday. Watch videos 2.5 and 2.6 for next Wednesday.

Proof exercise

Exercise

Prove: The sum of any two odd numbers is even (i.e. $\forall x, y \in \mathbb{R}$, x, y are odd $\implies x + y$ is even).

Order matters!

True or false:

1. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$
2. $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$

Proof exercise

Exercise

Prove $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$ is false.

Hint: Try to prove the negation (is true).

Homework

Prove $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$ (is true).

Standard induction

Standard induction

We have statements S_n dependent on a positive integer n. To prove $\forall n \in \mathbb{Z}_+$, S_n is true using standard induction we need to prove the following two statements:

1. Base case: S_1 is true.
2. Induction step: "$\forall n \geq 1$, $S_n \implies S_{n+1}$" is true.

For example, S_n can be the statement "$1 + 2 + ... + n = \frac{(n)(n+1)}{2}$".

What if?

If you managed to show the following instead, what can you prove?

Non-standard induction 1

1. S_3 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+1}$" is true.

Non-standard induction 2

1. S_1 is true.
2. "$\forall n > 1,\ S_n \implies S_{n+1}$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+3}$" is true.

Nonstandard induction 4

1. S_7 is true.
2. "$\forall n > 3,\ S_{n+1} \implies S_n$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0,\ S_n \implies S_{n+3}$" is true.

What else do I need to prove to show S_n is true for all positive integers n?

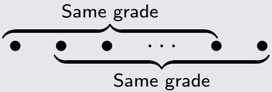
All 0s?

Theorem

$\forall N \in \mathbb{N}$, in every set of N MAT137 students, those students will get the same grade.

Proof.

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**
  Assume it is true for N. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ students. By induction hypothesis:
  - The first N students have the same grade.
  - The last N students have the same grade.



Hence the $N + 1$ students all have the same grade as well.

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