

- Topic: Negation, conditionals, definitions, and proofs
- **Homework:** Watch videos 1.14 and 1.15 for Wednesday.

Negate the following statements.

- 1 Every student at UofT has a cellphone.
- 2 There is a country in the European Union with fewer than 1000 inhabitants.
- 3 I like math and physics.
- 4 Everyone in this class likes math and physics .

Negation, a harder example

Negation example

Negate “Every page in this book contains at least one word whose first and last letters both come alphabetically before M”.

Hint: Try re-writing this sentence with a clause for each quantifier. For example, re-write this sentence starting with “For every page in this book, ...”. After you do this, negate systematically.

Am I lying?

I tell you: “If you get 80% or more on your first term test, then I will give you a piece of chocolate.”

In which of the following scenarios would I have lied (i.e. said something false)?

- 1 You get 80% on your test, and I give you a piece of chocolate.
- 2 You get 70% on your test, and I don't give you a piece of chocolate.
- 3 You get 100% on your test, and I don't give you a piece of chocolate.
- 4 You get 60% on your test, and I give you a piece of chocolate.
- 5 I give everybody a piece of chocolate after the marks are out.
- 6 You get 60% on your test.

My cards are on the table

Every card on the table has a number on one side and a letter on the other side.

I tell you: “(For all the cards on the table.) If a card has a vowel on one side then it must have an even number on the other side.”

You see 4 cards with “B”, “7”, “8”, “A”.

Which cards do you have to turn over to make sure I’m telling the truth?

My cards are on the table

What is the negation of the statement:

“(For all the cards on the table.) If a card has a vowel on one side then it must have an even number on the other side.”

- 1 “If a card has a vowel on one side then it must have an odd number on the other side.”
- 2 “If a card has a consonant on one side then it must have an even number on the other side.”
- 3 “If a card has a consonant on one side then it must have an odd number on the other side.”
- 4 “There is a card with a vowel on one side and an odd number on the other side.”

True or false?

① $\forall x \in \mathbb{R}, x > 0 \implies x \geq 0.$

② $\forall x \in \mathbb{R}, x \geq 0 \implies x > 0.$

Even and Odd

For $x \in \mathbb{R}$, give a mathematical definition for the statements “ x is an even number”. Do the same for the statement “ x is an odd number”.

A theorem about odd and even numbers

Claim

The sum of two odd numbers is even.

This should be interpreted as “The sum of any two odd numbers is even.”

Claim

$\forall x, y \in \mathbb{R}, x, y \text{ are odd} \implies x + y \text{ is even.}$

Claim

The sum of any two odd numbers is even.

Proof

1 is odd.

3 is odd.

$1 + 3 = 4$ is even.

Claim

The sum of any two odd numbers is even.

Proof

For all n :

$$\text{EVEN} + \text{EVEN} = \text{EVEN}$$

$$\text{EVEN} + \text{ODD} = \text{ODD}$$

$$\text{ODD} + \text{ODD} = \text{EVEN}$$

Claim

The sum of any two odd numbers is even.

Proof

$(2a+1)+(2b+1)=2a+2b+2$ is even.

Exercise

Prove: The sum of any two odd numbers is even (i.e. $\forall x, y \in \mathbb{R}, x, y \text{ are odd} \implies x + y \text{ is even}$).

Order matters!

True or false:

① $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ s.t. } x + y = 0$

② $\exists y \in \mathbb{R} \text{ s.t. } \forall x \in \mathbb{R}, x + y = 0$

Exercise

Prove $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$ is false.

Hint: Try to prove the negation (is true).

Homework

Prove $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$ (is true).