Today's topics and news

- Topic: Taylor series applications

Taylor series gymnastics

Write the following functions in a way where you can easily find their Maclaurin series using series you already know.

1. $f(x) = \dfrac{x^2}{1+x}$
2. $f(x) = (e^x)^2$
3. $f(x) = \sin\left(2x^3\right)$
4. $f(x) = \cos^2 x$
5. $f(x) = \ln \dfrac{1+x}{1-x}$
6. $f(x) = \dfrac{1}{(1+x^2)(1+x)}$

Note: You do not need to take any derivatives. You can reduce them all to other Maclaurin series you know.

Add these series

1. $$\sum_{n=2}^{\infty} \frac{(-2)^n}{(2n+1)!}$$

2. $$\sum_{n=0}^{\infty} (4n+1)x^{4n+2}$$

3. $$\sum_{n=0}^{\infty} \frac{2^n}{(2n)!}$$

4. $$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!(n+1)}$$

Add these series

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(-2)^n}{(2n+1)!}$ *Hint:* Think of sin

2. $\displaystyle\sum_{n=0}^{\infty} (4n+1)x^{4n+2}$ *Hint:* $\dfrac{d}{dx}\left[x^{4n+1}\right] = ???$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{(2n)!}$ *Hint:* Write the first few terms. Combine e^x and e^{-x}

4. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!(n+1)}$ *Hint:* Integrate

Limits

Compute these limits by writing out the first few terms of the Maclaurin series of numerator and denominator:

1. $$\lim_{x\to 0} \frac{\sin x - x}{x^3}$$
2. $$\lim_{x\to 0} \frac{6\sin x - 6x + x^3}{x^5}$$
3. $$\lim_{x\to 0} \frac{\cos(x) - 1 + \frac{1}{2}x\sin(x)}{\ln(1+x)^4}$$

Limits

Find a value of $a \in \mathbb{R}$ such that the limit

$$\lim_{x \to 0} \frac{e^{\sin x} - e^x + ax^3}{x^4}$$

exists and is not 0. Then compute the limit.

Outroduction