Today's topics and news

- Topic: New power series, Taylor series applications
- **Homework:** Nothing! Just come enjoy the show.

Power series warm-up

Suppose $\sum_{n=0}^{\infty}[a_n(x-1)^n]$ has an interval of convergence of (0,2]. Write the interval of convergence of the following series.

1. $\sum_{n=0}^{\infty}[2a_n(x-1)^n]$.
2. $\sum_{n=0}^{\infty}[a_n x^n]$.
3. $\sum_{n=0}^{\infty}[\frac{(-1)^n}{2^n}a_n(x-1)^n]$.
4. $\sum_{n=0}^{\infty}[a_n(x-1)^{2n}]$.

Review: what's wrong with the following computation?

Let us find the Taylor series for $\cos(x)$ with the Taylor series for $\sin(x)$. We know $\cos(x) = \int \sin(x)dx$ so:

$$\cos(x) = \int \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} dx$$
$$= \sum_{n=0}^{\infty} \int \frac{(-1)^n x^{2n+1}}{(2n+1)!} dx$$
$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+2}}{(2n+2)!}$$

Fix this computation.

Arctan

1. Write the function

$$f(x) = \arctan x$$

as a power series centered at 0.
Hint: Compute the first derivative. Then stop to think.

2. What is $f^{(2019)}(0)$?

Maclaurin series and derivatives at 0

Let f be an analytic function defined on some interval centred at 0. We define a new function g via the equation $g(x) = f(x^2)$.

Find $g^{(n)}(0)$ in terms of the derivatives of f at 0.

Hint: Write a Maclaurin series for g in two different ways.

Integrals

Consider the function

$$F(x) = \int_0^x \frac{\sin t}{t}\, dt.$$

It is not possible to find an elementary antiderivative.

1. Write $F(x)$ as a power series.
2. Estimate $F(1)$ with an error smaller than 0.01. (Hint: How do you bound the error of an alternating series?)