Today's topics and news

- Topic: Taylor polynomials
- **Homework:** Watch videos 14.5 - 14.8 for next Tuesday and 14.9, 14.10 for next Wednesday.

What can you conclude?

Think of the power series $\sum_n a_n x^n$. Do not assume $a_n \geq 0$.

In each case, may the given series be absolutely convergent (AC)? conditionally convergent (CC)? divergent (D)? all of them?

IF	$\sum_n a_n 3^n$ is ...	AC	CC	D
THEN	$\sum_n a_n 2^n$ may be ...	???	???	???
	$\sum_n a_n (-3)^n$ may be ...	???	???	???
	$\sum_n a_n 4^n$ may be ...	???	???	???

Different types of convergence on the boundary of IC

In the previous slide, we ruled out several combinations of convergence and divergence behaviour that can happen on the boundary of IC. We are going to now see that the rest is possible. Match each power series with what we know about its convergence:

1. IC $= [-1, 1]$, AC at -1, and AC at 1
2. IC $= [-1, 1]$, CC at -1, and CC at 1
3. IC $= (-1, 1]$
4. IC $= [-1, 1)$
5. IC $= (-1, 1)$
6. IC $= \mathbb{R}$
7. IC $= \{0\}$

a. $\sum_n n! x^n$
b. $\sum_n \frac{1}{n} x^n$
c. $\sum_n \frac{1}{n^2} x^n$
d. $\sum_n \frac{1}{n!} x^n$
e. $\sum_n x^n$
f. $\sum_n \frac{(-1)^n}{n} x^n$
g. $\sum \frac{(-1)^n}{n^2} x^{2n}$

Different centres and radii of convergence

By changing the previous examples (c), give a power series which satisfies the following:

1. IC $= [0, 2]$
2. IC $= [-2, 2]$
3. IC $= [-1, 0]$

Hint: For 2, change the coefficients of (c) a little.

The definitions of Taylor polynomial

Let f be a function defined at and near $a \in \mathbb{R}$. Let $n \in \mathbb{N}$.
Let P_n be the n-th Taylor polynomial for f at a.
Which ones of these is true?

1. P_n is an approximation for f of order n near a.
2. f is an approximation for P_n of order n near a.
3. $\lim_{x \to a} [f(x) - P_n(x)] = 0$
4. $\lim_{x \to a} \frac{f(x) - P_n(x)}{(x-a)^n} = 0$
5. $\exists$ a function R_n s.t. $f(x) = P_n(x) + R_n(x)$ and $\lim_{x \to a} \frac{R_n(x)}{(x-a)^n} = 0$
6. $f^{(n)}(a) = P_n^{(n)}(a)$
7. $\forall k = 0, 1, 2, \ldots, n, \quad f^{(k)}(a) = P_n^{(k)}(a)$
8. If x is close to a, then $f(x) = P_n(x)$.

True or False – C^n functions

1. If f and g are C^n functions, then $f+g$ is a C^n function.
2. f is continuous if and only if f is C^0.
3. f is differentiable if and only if f is C^1.
4. If f is a C^n function, then f is a C^{n-1} function.
5. If f is C^n at a, then f is C^n on some interval centered at a.
6. If f is C^n at a, then f is C^{n-1} on some interval centered at a.