Today's topics and news

- Topic: Ratio test, Power series
- **Homework:** Watch videos 14.3, 14.4 for Wednesday.

List of tests

We have learned:

- Divergence test
- Integral test
- BCT
- LCT
- Alternating series test
- Absolute convergence test

Today we will talk about:

- Ratio test

Positive and negative terms

- Let $\sum a_n$ be a series.
- Call $\sum$(P.T.) the sum of only the positive terms of the same series.
- Call $\sum$(N.T.) the sum of only the negative terms of the same series.

	$\sum$(P.T.) may be...	$\sum$(N.T.) may be...
In general		
If $\sum a_n$ is CONV		
If $\sum \lvert a_n \rvert$ is CONV		
If $\sum a_n$ is ABS CONV		
If $\sum a_n$ is COND CONV		
If $\sum a_n = \infty$		
If $\sum a_n$ is DIV (not to ∞ or $-\infty$)		

The inconclusive case of the ratio test

Show the ratio test is inconclusive when $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$.

Ratio test: Convergent or divergent?

Use Ratio test to decide which series are convergent:

1. $$\sum_{n=1}^{\infty} \frac{3^n}{n!}$$
2. $$\sum_{n=1}^{\infty} \frac{(2n)!}{n!^2\, 3^{n+1}}$$
3. $$\sum_{n=2}^{\infty} \frac{1}{\ln n}$$
4. $$\sum_{n=2}^{\infty} \frac{n!}{n^n}$$

Root test

Here is a new convergence test.

Theorem

Let $\sum_n a_n$ be a series. Assume the limit $L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$ exists.

- IF $0 \leq L < 1$ THEN the series is ???
- IF $L > 1$ THEN the series is ???

Without writing an actual proof, guess the conclusion of the theorem. Now write a proof for your conclusion in the case $L = \frac{1}{2}$ by doing a comparison with a reasonable series.

Hint: Imitate the explanation on Video 13.18 for the Ratio Test. For large values of n, what can you compare $|a_n|$ to? Start with the simplest series $\sum_n b_n$ s.t. $\lim_{n\to\infty} |b_n|^{\frac{1}{n}} = L$. Change it a little.

Power Series

A power series is a series (dependent on some variable, ex. x) of the form
$\sum_{n=0}^{\infty} c_n(x-a)^n$

In this form, we say the power series is centred at a.

A power series isn't actually a series because the terms aren't numbers. However, if I plug in a specific number for x, then it becomes a genuine series. Notice a power series always converges to c_0 if I plug in $x = a$.

Depending on what x you plug in, the power series might be convergent or divergent. On the x-values where the power series converges, you can think of the power series as representing some function $f(x)$.

Geometric series

Example: Consider the geometric power series $\sum_{n=0}^{\infty} x^n$. It converges iff $|x| < 1$. On this interval, the power series as a function is equal to the the function $\frac{1}{1-x}$.

Interval of convergence

Find the interval of convergence of each power series:

1. $$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$
2. $$\sum_{n=1}^{\infty} \frac{(x-5)^n}{n^2\, 2^{2n+1}}$$
3. $$\sum_{n=1}^{\infty} \frac{n^n}{42^n} x^n$$
4. (Hard!) $$\sum_{n=0}^{\infty} \frac{(3n)!}{n!(2n)!}\, x^n$$

What can you conclude?

Think of the power series $\sum_n a_n x^n$. Do not assume $a_n \geq 0$.

In each case, may the given series be absolutely convergent (AC)? conditionally convergent (CC)? divergent (D)? all of them?

IF	$\sum_n a_n 3^n$ is ...	AC	CC	D
THEN	$\sum_n a_n 2^n$ may be ...	???	???	???
	$\sum_n a_n (-3)^n$ may be ...	???	???	???
	$\sum_n a_n 4^n$ may be ...	???	???	???