Today's topics and news

- Topic: Alternating series, absolute and conditional convergence
- **Homework:** Watch videos 13.18, 13.19, 14.1, 14.2 for Tuesday and videos 14.3, 14.4 for Wednesday.
- **Test 4** takes place next Tuesday. It will cover up to and including today's lecture/videos. In particular, the ratio test (13.18, 13.19) will not be on the test so you will only need to know 6 tests.

List of tests

We have learned:

- Divergence test
- Integral test
- BCT
- LCT

Today we will talk about:

- Alternating series test
- Absolute convergence test

An AST example

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty}(-1)^n\frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

Estimation

Estimate the sum

$$S = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

with an error smaller than 0.001.

True or False – Absolute Values

1. IF $\{a_n\}_{n=1}^{\infty}$ is convergent, THEN $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent.
2. IF $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent, THEN $\{a_n\}_{n=1}^{\infty}$ is convergent.
3. IF $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent, THEN $\displaystyle\sum_{n=1}^{\infty} |a_n|$ is convergent.
4. IF $\displaystyle\sum_{n=1}^{\infty} |a_n|$ is convergent, THEN $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

Example

Does $\sum_n \frac{\sin(n)}{n^2}$ converge or diverge?

Positive and negative terms

- Let $\sum a_n$ be a series.
- Call $\sum$(P.T.) the sum of only the positive terms of the same series.
- Call $\sum$(N.T.) the sum of only the negative terms of the same series.

IF $\sum$(P.T.) is...	AND $\sum$(N.T.) is...	THEN $\sum a_n$ may be...
CONV	CONV	
∞	CONV	
CONV	$-\infty$	
∞	$-\infty$	

Positive and negative terms

- Let $\sum a_n$ be a series.
- Call $\sum$(P.T.) the sum of only the positive terms of the same series.
- Call $\sum$(N.T.) the sum of only the negative terms of the same series.

	$\sum$(P.T.) may be...	$\sum$(N.T.) may be...
In general		
If $\sum a_n$ is CONV		
If $\sum \lvert a_n \rvert$ is CONV		
If $\sum a_n$ is ABS CONV		
If $\sum a_n$ is COND CONV		
If $\sum a_n = \infty$		
If $\sum a_n$ is DIV (not to ∞ or $-\infty$)		