Today's topics and news

- Topic: Integral test, BCT, LCT for series, and alternating series
- **Homework:** Watch videos 13.15 - 13.17 for Wednesday.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

9. IF $\lim_{n\to\infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
10. IF $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
11. IF $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n \right] = 0$.
12. IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

    THEN $\sum_{n=0}^{\infty} a_n$ is convergent.

True or false

Assume $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ converges in the following.

1. $$\sum_{n=0}^{\infty}(a_n + cb_n) = \sum_{n=0}^{\infty} a_n + \text{c}\sum_{n=0}^{\infty} b_n$$
2. $$\sum_{n=0}^{\infty}(a_n b_n) = \left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right)$$
3. IF $a_n \leq c_n \leq b_n$ and $\sum_{n=0}^{\infty} a_n$, $\sum_{n=0}^{\infty} b_n$ both converge THEN $\sum_{n=0}^{\infty} c_n$ converges.

List of tests

We have learned:

- Divergence test (WARNING: This can only tell you if a series diverges. It will never check if a series converges.)

Today we will talk about:

- Integral test
- BCT
- LCT
- Alternating series test

Rapid questions: For which values of $p \in \mathbb{R}$ are these series convergent? What does the series converge to?

1. $$\sum_{n=1}^{\infty} \frac{1}{p^n}$$
2. $$\sum_{n=1}^{\infty} \frac{1}{n^p}$$
3. $$\sum_{n=1}^{\infty} p^n$$
4. $$\sum_{n=1}^{\infty} n^p$$

More rapid questions: Convergent or divergent?

1. $$\sum_{n}^{\infty} \frac{n^{10} + 17n^7 + 3}{n^{11}}$$

2. $$\sum_{n}^{\infty} \frac{\sqrt[3]{n^2+1}+1}{\sqrt{n^4+n}+n+1}$$

Slower questions: convergent or divergent?

Using the tests you've learned so far, check if the following converges or diverges. You do not need to write out your solution formally for this exercise.

1. $\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$
2. $\sum_{n}^{\infty} \frac{1}{n \ln n}$
3. $\sum_{n}^{\infty} \sin \frac{1}{n^2}$
4. $\sum_{n}^{\infty} \frac{1}{n(\ln n)^3}$
5. $\sum_{n}^{\infty} \frac{(\ln n)^{20}}{n^2}$
6. $\sum_{n}^{\infty} e^{-n^2}$

New series based on a convergent series

We know

- $\forall n \in \mathbb{N}$, $0 < a_n < 1$.
- the series $\displaystyle\sum_{n}^{\infty} a_n$ is convergent

Determine whether the following series are convergent, divergent, or we do not have enough information to decide:

1. $\displaystyle\sum_{n}^{\infty} \sin a_n$
2. $\displaystyle\sum_{n}^{\infty} \cos a_n$
3. $\displaystyle\sum_{n}^{\infty} \sqrt{a_n}$