Today's topics and news

- Topic: Properties of series, divergence test/necessary condition

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- **Homework:** Watch videos 13.10 - 13.14 for Tuesday and 13.15 - 13.17 for Wednesday.

Geometric series

Calculate the value of the following series:

1. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$
2. $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \dots$
3. $\frac{3}{2} - \frac{9}{4} + \frac{27}{8} - \frac{81}{16} + \dots$
4. $\sum_{n=k}^{\infty} x^n$

True or False – The Necessary Condition

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^\infty a_n$ is convergent.
2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_n^\infty a_n$ is divergent.
3. IF $\sum_n^\infty a_n$ is convergent THEN $\lim_{n\to\infty} a_n = 0$.
4. IF $\sum_n^\infty a_n$ is divergent THEN $\lim_{n\to\infty} a_n \neq 0$.

Warm-up

Does the series $\sum_{n=1}^{\infty} n \sin(\frac{1}{n})$ converge or diverge?

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Does the series $\sum_{n=1}^{\infty} n \sin(\frac{1}{n})$ converge or diverge?

1. Divergence test/necessary condition is the first convergence test you have learned for series. We will learn 6 more (depending on how you count). Keep the list in your mind.
2. There is a similar statement to the necessary condition for type 1 improper integrals as well: Let f be continuous on $[a, \infty)$. IF $\int_a^{\infty} f(x)dx$ converges THEN $\lim_{x \to \infty} f(x) = 0$ or DNE. (Note you cannot rule out the limit DNE unlike the case for series.)

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$
2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$
3. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.
4. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

6. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing, THEN $\forall n \geq 0$, $a_n > 0$.

7. IF $\lim_{n\to\infty} a_n = 0$, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

8. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.