Today's topics and news

- Topic: Limit comparison for improper integrals, Series
- **Homework:** Watch videos 13.8 and 13.9 for Wednesday.

Rapid questions: review of improper integrals

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$
4. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$
5. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$
6. $\displaystyle\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$
8. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\displaystyle\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

Convergent or divergent?

1. $\displaystyle\int_1^\infty \frac{x^3+2x+7}{x^5+11x^4+1}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^2+x+1}}\,dx$
3. $\displaystyle\int_0^1 \frac{3\cos x}{x+\sqrt{x}}\,dx$
4. $\displaystyle\int_0^1 \cot x\,dx$
5. $\displaystyle\int_0^1 \frac{\sin x}{x^{3/2}}\,dx$
6. $\displaystyle\int_0^\infty e^{-x^2}\,dx$
7. $\displaystyle\int_2^\infty \frac{(\ln x)^{10}}{x^2}\,dx$
8. $\displaystyle\int_0^1 \frac{\arcsin x}{x^{3/2}}\,dx$

Convergent or divergent?

5. $$\int_0^1 \frac{\sin x}{x^{3/2}}\,dx$$
6. $$\int_0^\infty e^{-x^2}\,dx$$
7. $$\int_2^\infty \frac{(\ln x)^{10}}{x^2}\,dx$$
8. $$\int_0^1 \frac{\arcsin x}{x^{3/2}}\,dx$$

Using LCT

Does the following improper integral converge or diverge?

$$\int_{1}^{\infty} \frac{1}{\sqrt{x-1}(x+2)} dx$$

What is wrong with this calculation? Fix it

Claim:

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

“Justification”

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \ldots) - (\ln 3 + \ln 4 + \ldots) \\
&= \ln 2
\end{aligned}$$

Trig series: convergent or divergent?

1. $$\sum_{n=0}^{\infty} \sin(n\pi)$$
2. $$\sum_{n=0}^{\infty} \cos(n\pi)$$

Hint: Compute the partial sums.

A telescopic series

I want to calculate the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$.

1. Find a formula for the k-th partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^2 + 2n}$.

   Hint: Write $\frac{1}{n^2 + n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$$

Challenge: Compute $S = \sum_{n=2}^{\infty} \frac{3 - 5n}{n^3 - n}$.

A telescopic series

Calculate the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$.

Harmonic series

For each $n > 0$, we define r_n = smallest number of form 2^k (where $k \in \mathbb{N}$) that is greater than or equal to n,

Consider the series $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$

1. Compute r_1 through r_8
2. Compute the partial sums S_1, S_2, S_4, S_8 for the series S.
3. Calculate $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.
4. Calculate $H = \sum_{n=1}^{\infty} \frac{1}{n}$.

   Hint: "Compare" H and S.