Today's topics and news

- Topic: Volume, sequences
- **Homework:** Watch videos 11.3 - 11.6 for Tuesday and 11.7, 11.8 for Wednesday. (11.7 is probably the single most applicable/useful theorem in the last quarter of the course)

Doghnut

Let R be the region inside the curve with equation

$$(x-4)^2 + y^2 = 1.$$

Rotate R around the line with equation $x = 1$. The resulting solid is a *solid torus*.

1. Draw a picture and convince yourself that a torus looks like a doughnut.
2. Compute the volume of the torus as an integral using the cylindrical shell method.
3. Compute the volume of the torus as an integral using the slicing method.

Fill in which method would apply

	dx	dy
Region in x-y plane rotated about x-axis	?	?
Region in x-y plane rotated about y-axis	?	?

Note: Depending on whether your region is described by functions of x or functions of y, one of the two choices of dx or dy might be better.

Challenge

Two cylinders (of infinite length) have the same radius R and their axes meet at a right angle. Find the volume of their intersection.

Hint: You can slice the resulting solid by parallel cuts in three different directions. One of the three makes the problem much, much simpler than the other two.

Warm up

Write a formula for the general term of these sequences

1. $\{a_n\}_{n=0}^{\infty} = \{1, 4, 9, 16, 25, \dots\}$
2. $\{b_n\}_{n=1}^{\infty} = \{1, -2, 4, -8, 16, -32, \dots\}$
3. $\{c_n\}_{n=1}^{\infty} = \left\{\frac{2}{1!}, \frac{3}{2!}, \frac{4}{3!}, \frac{5}{4!}, \dots\right\}$
4. $\{d_n\}_{n=1}^{\infty} = \{1, 4, 7, 10, 13, \dots\}$

True or False?

Let f be a function with domain at least $[1, \infty)$.
We define a sequence as $a_n = f(n)$.
Let $L \in \mathbb{R}$.

1. IF $\lim_{x \to \infty} f(x) = L$, THEN $\lim_{n \to \infty} a_n = L$.

2. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{x \to \infty} f(x) = L$.

3. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{n \to \infty} a_{n+1} = L$.

Definition of limit of a sequence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

1. $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
2. $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n > n_0 \implies |L - a_n| < \varepsilon.$
3. $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{R}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
4. $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| \leq \varepsilon.$
5. $\forall \varepsilon \in (0, 1), \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \varepsilon.$
6. $\forall \varepsilon > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \frac{1}{\varepsilon}.$
7. $\forall k \in \mathbb{Z}^+ > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < k.$
8. $\forall k \in \mathbb{Z}^+ > 0, \ \exists n_0 \in \mathbb{N}$ s.t. $\forall n \in \mathbb{N}, \quad n \geq n_0 \implies |L - a_n| < \frac{1}{k}.$