Today's topics and news

- Topic: Integration of rational functions wrap-up and volumes
- **Homework:** Watch videos 11.1 and 11.2 for Wednesday.

What we've done so far on integration of rational functions

Based on what we did in the slides on Tuesday. We learned that

1. For $a \neq b$ and polynomial $P(x)$, $\frac{P(x)}{(x-a)(x-b)}$ can always be wrriten as $Q(x) + \frac{A}{x-a} + \frac{B}{x-b}$, where $Q(x)$ is a polynomial and A and B are real numbers.
2. For $a \in \mathbb{R}$ and polynomial $P(x)$, $\frac{P(x)}{(x-a)^2}$ can always be written as $Q(x) + \frac{A}{x-a} + \frac{B}{(x-a)^2}$, where $Q(x)$ is a polynomial and A and B are real numbers.

Explain to a partner how you would integrate an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)} dx\ ?$$

Specifically, how it should be decomposed before you try to integrate it.

Irreducible quadratics

1. Calculate $\int \frac{1}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$.
   Hint: You should be able to do these very quickly.
2. Calculate $\int \frac{2x+3}{x^2+1}\,dx$
3. Calculate $\int \frac{x^3}{x^2+1}\,dx$
4. Calculate $\int \frac{x}{x^2+x+1}\,dx$

   Hint: Transform it into one like the previous ones

Messier rational functions

How can we compute an integral of the form

$$\int \frac{\text{polynomial}}{(x+1)^3(x+2)x^4(x^2+1)(x^2+4x+7)}dx\ ?$$

Pyramid

Compute the volume of a pyramid with height H and square base with side length L.

Easy version: Assume the top of the pyramid is on top of the centre of the square base.

Hard version: Assume the top of the pyramid is not on top of the centre of the square base. (This is sometimes called an oblique pyramid.)

Hint: Slice the pyramid like a carrot with cuts parallel to the base. Think about the shape of the dimension of these slices.

An equation for volumes by “slicing”

Let $0 < a < b$.
Let f be a continuous, positive function defined on $[a, b]$.
Let R be the region in the first quadrant bounded between the graph of f and the x-axis.

Revolve R around the x-axis, what shape does the line over each x-value become? What is the area of this shape?

Find a formula for the volume of the solid of revolution obtained by rotation the region R around the x-axis.

An equation for volumes by "cylindrical shells"

Let $0 < a < b$.
Let f be a continuous, positive function defined on $[a, b]$.
Let R be the region in the first quadrant bounded between the graph of f and the x-axis.

Revolve R around the y-axis, what shape does the line over each x-value become? What is the area of this shape?

Find a formula for the volume of the solid of revolution obtained by rotation the region R around the y-axis.

Many axis of rotation

Let R be the region in the first quadrant bounded between the curves with equations $y = x^3$ and $y = \sqrt{32x}$. Compute the volume of the solid of revolution obtained by rotating R around...

1. ... the x-axis using both methods
2. ... the line $y = -1$ using either methods
3. ... the y-axis using either methods