Today's topics and news

- Topics: Conditionals and proofs
- **Homework for Tuesday:** Watch video 2.4 and write a proof to the claims on the last slide.

Am I lying?

I tell you:

"If you get 80% or more on your first term test, I will give you a piece of chocolate."

In which of the following scenarios would I have lied?

1. You get 80% on your test, and I give you a piece of chocolate.
2. You get 70% on your test, and I don't give you a piece of chocolate.
3. You get 100% on your test, and I don't give you a piece of chocolate.
4. You get 60% on your test, and I give you a piece of chocolate.
5. I give everybody a piece of chocolate after the marks are out.
6. You get 60% on your test.

Negate the sentence.

Cards

Every card on the table has a number on one side and a letter on the other side.

I tell you:

"If a card has a vowel on one side then it must have an even number on the other side."

You see 4 cards with "B", "7", "8", "A".

Which cards do you have to turn over to make sure I'm telling the truth?

Answer: "A" and "7" What's the negation of this sentence?

Equivalence

Which of the following statements are equivalent to:

"Every MAT137 student likes math"

1. "(For all students,) if a student is in MAT137, then they likes math."
2. "(For all students,) if a student likes math, then they are in MAT137."
3. "(For all students,) if a student dislikes math, then they are not in MAT137."
4. "(For all students,) if a student is not in MAT137, then they like math."
5. "(For all students,) if a student is not in MAT137, then they dislike math."
6. "There is a student in MAT137 who likes math."
7. "There is a student in MAT137 who dislikes math."

Equivalence

Which of the following statements are equivalent to:

"Every MAT137 student likes math"

1. "(For all students,) if a student likes math, then they are in MAT137."
2. "(For all students,) if a student is in MAT137, then they likes math."
3. "(For all students,) if a student dislikes math, then they are not in MAT137."
4. "(For all students,) if a student is not in MAT137, then they like math."
5. "(For all students,) if a student is not in MAT137, then they dislike math."
6. "There is a student in MAT137 who likes math."
7. "There is a student in MAT137 who dislikes math."

Definition

Even and Odd

For $x \in \mathbb{R}$, give a mathematical definition for the statements "x is an even number". Do the same for the statement "x is an odd number".

Proof?

Claim

The sum of two odd numbers is even.

This should be interpreted as "The sum of any two odd numbers is even.

Claim

$\forall x, y \in \mathbb{R}$, x, y are odd $\implies$ $x + y$ is even.

Proof?

Claim

The sum of any two odd numbers is even.

Proof

1 is odd.

3 is odd.

$1 + 3 = 4$ is even.

Proof?

Claim

The sum of any two odd numbers is even.

Proof

$(2a+1)+(2b+1)=2a+2b+2$ is even.

Proof Practice

Exercise

Prove: The sum of any two odd numbers is even.

Proof exercise

Exercise

Prove or disprove: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $x + y = 0$.

Proof exercise

Exercise

Prove or disprove: $\exists y \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, x + y = 0$.

Hint: Try to prove the negation.

Induction

Induction

We have statements S_n dependent on a positive integer n. To prove S_n are all true using standard induction we need to prove the two facts:

1. S_1 is true.
2. "For all $n \geq 1$, if S_n then S_{n+1}" is true.

Explain intuitively why proving these two statements would prove all the statements S_n are true.

What if?

If you managed to show the following instead, what can you prove?

Non-standard induction 1

1. S_3 is true.
2. "$\forall n > 0$, $S_n \implies S_{n+1}$" is true.

Non-standard induction 2

1. S_1 is true.
2. "$\forall n > 1$, $S_n \implies S_{n+1}$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0$, $S_n \implies S_{n+3}$" is true.

Nonstandard induction 4

1. S_7 is true.
2. "$\forall n > 3$, $S_{n+1} \implies S_n$" is true.

What if?

Non-standard induction 3

1. S_1 is true.
2. "$\forall n > 0$, $S_n \implies S_{n+3}$" is true.

What else do I need to prove to show S_n is true for all positive integers n?

Proof exercise

For all $A \subseteq \mathbb{R}$.

- Ivan likes A if and only if $\exists a \in \mathbb{R}$ s.t. $\forall x \in A, x \leq a$.
- Alfonso likes A if and only if $\exists a \in \mathbb{R}$ s.t. $\forall x \in A, x < a$.

Decide if the following statements are true or false. Prove your assertion.

1. For every $A \subseteq \mathbb{R}$, if Alfonso likes A then Ivan likes A.
2. For every $A \subseteq \mathbb{R}$, if Ivan likes A then Alfonso likes A.