MAT 137
Tutorial #17– Convergence Tests
August 6/7, 2019

Determine which ones of the following series are absolutely convergent, conditionally convergent, or divergent.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{e^{1-1/n}}{3+\sin n}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+6}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{\sqrt{n^5+4n+11}}$

7. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \sin\frac{1}{n}$

8. $\displaystyle\sum_{n=1}^{\infty} (-1)^n n \sin\frac{1}{n}$

9. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+3)2^n}{n!}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{n!(2n)!}{(3n)!}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdot \ldots \cdot (2n)}{1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)} \cdot \frac{\pi^{n+1}}{e^{2n-1}}$

13. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

14. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{10^{4n}}$

15. $\displaystyle\frac{1}{2}+\frac{2}{3^2}-\frac{4}{4^3}+\frac{8}{5^4}+\frac{16}{6^5}-\frac{32}{7^6}+\ldots$

16. $\displaystyle\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{\ln n}$

17. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

18. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\ln n}$

19. $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^{1.1}}$

20. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^3}$

Summary of Convergence Tests for Series
(by Beatriz Navarro-Lameda and Nikita Nikolaev)

Test	When to Use	Conclusions
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$ if $\lvert r\rvert < 1$; diverges if $\lvert r\rvert \geq 1$.
Necessary Condition	All series	If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.
Integral Test	• $a_n = f(n)$ • f is continuous, positive and decreasing. • $\int_1^{\infty} f(x)\,dx$ is easy to compute.	$\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ both converge or both diverge.
p-series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges for $p > 1$; diverges for $p \leq 1$.
Basic Comparison Test	$0 \leq a_n \leq b_n$	If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges
Limit Comparison Test	$a_n, b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ $(0 < L < \infty)$	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^n b_n$, $b_n \geq 0$	If • $b_n > 0$, $\forall n$ • $\{b_n\}$ is decreasing • $\lim_{n\to\infty} b_n = 0$ Then $\sum_{n=1}^{\infty} (-1)^n b_n$ is convergent.
Absolute Convergence	Series with some positive terms and some negative terms (including alternating series)	If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ converges (absolutely).
Ratio Test	Any series (especially those involving exponentials and/or factorials)	For $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$ (including $L = \infty$), • If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely • If $L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges • If $L = 1$, then we can draw no conclusion.