

**MAT 137**  
**Tutorial #15– Improper Integrals**  
**July 29/30, 2019**

1. Determine whether the following integrals are convergent or divergent directly from the definition of improper integral. If they are convergent, calculate their value.

(a)  $\int_{-1}^{\infty} \frac{1}{x^2 + 1} dx$

(e)  $\int_0^{\infty} \cos x dx$

(b)  $\int_2^{\infty} \frac{1}{x + 2} dx$

(f)  $\int_0^1 \frac{dx}{x^2}$

(c)  $\int_0^1 \ln x dx$

(g)  $\int_0^1 \frac{dx}{\sqrt{x}}$

(d)  $\int_1^{\infty} \ln x dx$

(h)  $\int_2^{\infty} \frac{1}{x^2 - 1} dx$

*Hint:* For Question 1h, write  $\frac{1}{x^2 - 1} = \frac{A}{x - 1} + \frac{B}{x + 1}$ .

2. For which values of the constant  $p$  is the integral  $\int_1^{\infty} \frac{1}{x^p} dx$  convergent?
3. Using the Basic Comparison Test and/or the Limit Comparison Test, determine which ones of the following improper integrals are convergent or divergent. Do not calculate their value.

(a)  $\int_1^{\infty} \frac{\sin x + 2 \cos x + 10}{x^2} dx$

(d)  $\int_0^{\infty} \frac{\arctan x}{x^{1.1}} dx$

(b)  $\int_0^{\infty} \frac{x - 7}{x^2 + x + 5} dx$

(e)  $\int_0^1 \frac{\sin x}{x^{4/3}} dx$

(c)  $\int_{10}^{\infty} \frac{\sqrt{x - 6}}{3x^2 + 5x + 11} dx$

(f)  $\int_0^{\infty} e^{-x^2} dx$