Math 337 Winter 2010, Problem set 4
Due Fri. Mar. 19

It is recommended that you try all the problems. Hand in only those which are starred for grading. The hints which are given are just suggestions. You may find a different or better way of doing the problem.

§2.9 E*, F Thus we have shown that there are lots of transcendental numbers, even though it is difficult to give any particular examples. This is an example of a pure existence proof in mathematics, where an indirect method is used to show that something exists, without giving any idea of how to construct an example. In fact π and e are transcendental numbers.

(1) Let us say that the cardinality of A is less than or equal to that of B, written $|A| \leq |B|$, if there is an injection $f : A \to B$. Show that this is the case if and only if there is a surjection $g : B \to A$. Hint: if g is such a surjection define f by choosing, for each $a \in A$, an element of $g^{-1}\{a\}$ and calling it $f(a)$. Although it may seem obvious that such a choice can be made there is a mathematical axiom called the axiom of choice which permits us to do this. See for example: http://en.wikipedia.org/wiki/Axiom of choice

§2.9 I, J, H* Also show that $|X| \leq |P(X)|$. (There is an obvious injection.)

(2) * Show that $|\mathbb{R}^2| = |\mathbb{R}|$. (Hint: replace $\mathbb{R}$ by $\{0,1\}^{\mathbb{N}}$ and construct a bijection by interleaving a pair of binary sequences to form a single sequence.)

§3.1 A

§3.2 P a, c, j, m, n, s, t, v, w, x.

(3) * Also determine convergence for each the following, with justification.

$$\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2} \qquad \sum_{n=2}^{\infty} \frac{1}{n \log n \log(\log n)} \qquad \sum_{n=1}^{\infty} \frac{n^{2n}}{(2n)!}$$

§3.3 F

§4.3 L, M In these problems replace $\mathbb{R}^n$ by a general metric space.

§4.4 J a, b Construct a bijection between S and the set $T = \{0,1,2\}^{\mathbb{N}}$ of ternary sequences. Also show that $|S| = |\mathbb{R}|$ by showing that $|T| = |[0,1)|$. (Mimic the proof that $|\{0,1\}^{\mathbb{N}}| = |[0,1)|$ using ternary rather than binary expansions.)

§5.4 E*, F In these problems $G(f)$ is to be considered as a subset of $\mathbb{R}^{n+m}$.

§5.5 J (The hint given is not necessarily the best approach.)

§6.3 N*

§6.4 C (This is a consequence of the intermediate value theorem for continuous functions.)