MAT 1000 2007-08
Prob. Set 1
Due Monday Oct. 15

All numbers refer to problems from Chapt. 1 of *Real analysis 3rd ed., G.B. Folland*. Some of the problems may resemble things done in lecture. Give full proofs nonetheless. **Hand in only the starred problems, including (2) in the "other problems" but make sure you can solve the others as well.**

3, 5*, 8* (see definition p. 2)
11*
12 Also prove that the metric is complete. (If $\{E_n\}$ is Cauchy show that $E_n \to \limsup E_n$)
#17, 18*, 22, 23, 24*, 26*, 28, 30*, 31*, 33*

Other problems.

(1) Let μ^* be the outer measure coming from a finite pre-measure μ on an algebra Show that the following are equvalent for a subset E of X.
   (a) E is μ^*-measurable in the sense of Caratheodory (see Folland for the definition).
   (b) $\mu^*(E) + \mu^*(E^c) = \mu^*(X)$.
   (c) For all $\epsilon > 0$ there is an $A \in \mathcal{A}$ such that $\mu^*(E \Delta A) < \epsilon$.

(2) * Suppose $E \in \mathcal{L}$, the Lebesgue σ-algebra of $\mathbb{R}$ and $\mu(E) < \infty$. Show that $\mu(E\Delta(E+t)) \to 0$ as $t \to 0$. (Do it first for finite unions of intervals and then use an approximation argument.) Deduce that for any non-null $E \in \mathcal{L}$ and integer $k > 0$ there exists $\delta > 0$ such that for any finite set $F \subset [0, \delta]$ of cardinality k there is an x such that $x + F \subset E$. (Show that the intersection of appropriate translates of E is non-empty.)